Universität Stuttgart

SimTech Cluster of Excellence

Benjamin Unger

Towards port-Hamiltonian systems with time-delays

joint work with T. Breiten and D. Hinsen

Trends on Dissipativity in Systems and Control
Brig, Switzerland
May 25, 2022

Earthquake engineering – hybrid testing

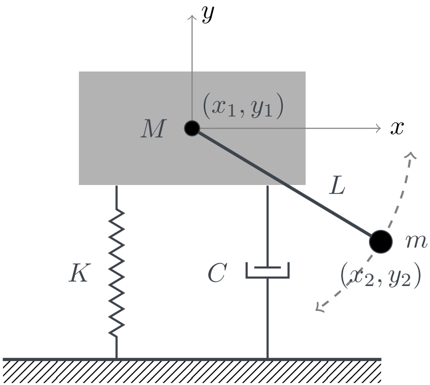
Earthquake engineering

Main goal: Make structures more resistant to earthquakes.

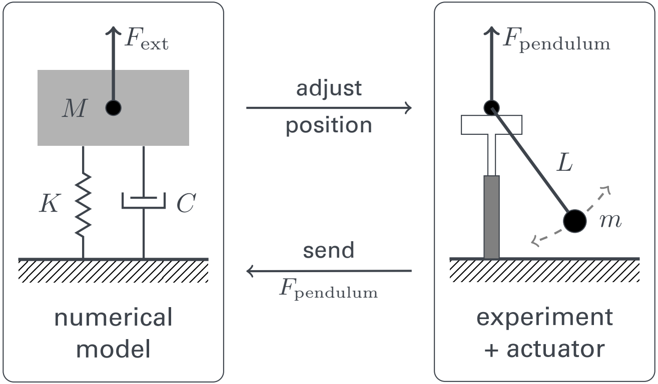
Problems:

- Highly complex system structures with many uncertain parameters
- Actual tests are extremely expensive

Earthquake engineering – hybrid testing



Earthquake engineering – hybrid testing



Time-delay systems

$$\begin{aligned} \dot{z}(t) &= A_0 z(t) + A_1 z(t-\tau) + Bu(t), \\ y(t) &= Cz(t) \end{aligned}$$

Time-delay systems

$$\dot{z}(t) = A_0 z(t) + A_1 z(t-\tau) + Bu(t),$$
$$y(t) = Cz(t)$$

Further motivation

- Delayed feedback/interconnection
- Transmission lines/propagation delay
- Hyperbolic equations

B. Unger.
Well-Posedness and Realization Theory for Delay Differential-Algebraic Equations
Dissertation, TU Berlin, 2020.

Time-delay systems

$$\dot{z}(t) = A_0 z(t) + A_1 z(t-\tau) + Bu(t),$$
$$y(t) = Cz(t)$$

Central question

What are port-Hamiltonian systems with delays?

PH and passivity

1

Port-Hamiltonian implies passivity

$$\dot{z}(t) = Az(t) + Bu(t), \qquad \dot{z}(t) = (J - R)Hz(t) + Bu(t),$$
$$y(t) = Cz(t), \qquad y(t) = B^\top Hz(t)$$

Definition

A system is called **passive** if there exists a state-dependent storage function $\mathcal{H}\colon \mathbb{R}^N \to \mathrm{R}_{\geq 0}$ such that the dissipation inequality

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{H}(z(t)) \leq \langle y(t), u(t)\rangle$$

is satisfied for any $t > 0$.

Passivity implies port-Hamiltonian

Theorem e.g. Beattie et al. '18

(Technical details aside.) The following are equivalent:

- The system is passive.
- The Kalman-Yakubovich-Popov (KYP) inequality

$$\mathcal{W}(H) := \begin{bmatrix} -A^\top H - HA & C^\top - HB \\ C - B^\top H & 0 \end{bmatrix} \geq 0$$

has a symmetric positive definite solution $H \in \mathbb{R}^{N \times N}$.

Passivity implies port-Hamiltonian

Theorem e.g. Beattie et al. '18

(Technical details aside.) The following are equivalent:

- The system is passive.
- The Kalman-Yakubovich-Popov (KYP) inequality

$$\mathcal{W}(H) := \begin{bmatrix} -A^\top H - HA & C^\top - HB \\ C - B^\top H & 0 \end{bmatrix} \geq 0$$

has a symmetric positive definite solution $H \in \mathbb{R}^{N \times N}$.

$$\begin{aligned} \dot{z}(t) &= Az(t) + Bu(t) \\ y(t) &= Cz(t) \end{aligned}$$

Passivity implies port-Hamiltonian

Theorem e.g. Beattie et al. '18

(Technical details aside.) The following are equivalent:

- The system is passive.
- The Kalman-Yakubovich-Popov (KYP) inequality

$$\mathcal{W}(H) := \begin{bmatrix} -A^\top H - HA & C^\top - HB \\ C - B^\top H & 0 \end{bmatrix} \geq 0$$

  has a symmetric positive definite solution $H \in \mathbb{R}^{N \times N}$.

$$\begin{aligned} H\dot{z}(t) &= HAz(t) + HBu(t) \\ y(t) &= \quad Cz(t) \end{aligned}$$

Passivity implies port-Hamiltonian

Theorem e.g. Beattie et al. '18

(Technical details aside.) The following are equivalent:

- The system is passive.
- The Kalman-Yakubovich-Popov (KYP) inequality

$$\mathcal{W}(H) := \begin{bmatrix} -A^\top H - HA & C^\top - HB \\ C - B^\top H & 0 \end{bmatrix} \geq 0$$

  has a symmetric positive definite solution $H \in \mathbb{R}^{N \times N}$.

$$\begin{aligned} H\dot{z}(t) &= HAz(t) + HBu(t) \\ y(t) &= B^\top Hz(t) \end{aligned}$$

Passivity implies port-Hamiltonian

Theorem — e.g. Beattie et al. '18

(Technical details aside.) The following are equivalent:

- The system is passive.
- The Kalman-Yakubovich-Popov (KYP) inequality

$$\mathcal{W}(H) := \begin{bmatrix} -A^\top H - HA & C^\top - HB \\ C - B^\top H & 0 \end{bmatrix} \geq 0$$

  has a symmetric positive definite solution $H \in \mathbb{R}^{N \times N}$.

$$\begin{aligned} H\dot{z}(t) &= HAz(t) + Gu(t) \\ y(t) &= G^\top z(t) \end{aligned}$$

Passivity implies port-Hamiltonian

Theorem — e.g. Beattie et al. '18

(Technical details aside.) The following are equivalent:

- The system is passive.
- The Kalman-Yakubovich-Popov (KYP) inequality

$$\mathcal{W}(H) := \begin{bmatrix} -A^\top H - HA & C^\top - HB \\ C - B^\top H & 0 \end{bmatrix} \geq 0$$

  has a symmetric positive definite solution $H \in \mathbb{R}^{N\times N}$.

$$\begin{aligned} H\dot{z}(t) &= \big(\mathrm{skew}(HA) - (-\mathrm{sym}(HA))\big)z(t) + Gu(t) \\ y(t) &= G^\top z(t) \end{aligned}$$

Passivity implies port-Hamiltonian

Theorem — e.g. Beattie et al. '18

(Technical details aside.) The following are equivalent:

- The system is passive.
- The Kalman-Yakubovich-Popov (KYP) inequality

$$\mathcal{W}(H) := \begin{bmatrix} -A^\top H - HA & C^\top - HB \\ C - B^\top H & 0 \end{bmatrix} \geq 0$$

  has a symmetric positive definite solution $H \in \mathbb{R}^{N \times N}$.

$$\begin{aligned} H\dot{z}(t) &= \big(J - R\big) z(t) + G u(t) \\ y(t) &= G^\top z(t) \end{aligned}$$

Passivity implies port-Hamiltonian

Theorem e.g. Beattie et al. '18

(Technical details a... ...quivalent:

- The system...
- The Kalm... ...lity

$$C^\top - HB \qquad 0 \quad] \geq 0$$

has a sy... ... $\in \mathbb{R}^{N\times N}$.

Equivalence

the pH structure provides a simple parameterization of passive systems

$$\ldots(t) + Gu(t)$$

$$y(t) = G^\top z(t)$$

PH and time-delays

2

A (very small) wish list for pH systems with time-delays

$$\begin{aligned}\dot{z}(t) &= A_0 z(t) + A_1 z(t-\tau) + Bu(t),\\ y(t) &= Cz(t)\end{aligned}$$

A (very small) wish list for pH systems with time-delays

$$\begin{aligned}\dot{z}(t) &= A_0 z(t) + A_1 z(t-\tau) + Bu(t),\\ y(t) &= Cz(t)\end{aligned}$$

Port-Hamiltonian formulation for time-delay systems (wish list)

- Hamiltonian is explicitly included in the system dynamics
- pH systems vs. passivity

A (very small) wish list for pH systems with time-delays

$$
\begin{aligned}
\dot{z}(t) &= A_0 z(t) + A_1 z(t-\tau) + Bu(t), \\
y(t) &= Cz(t)
\end{aligned}
$$

Port-Hamiltonian formulation for time-delay systems (wish list)

- Hamiltonian is explicitly included in the system dynamics
- pH systems vs. passivity

A (very small) wish list for pH systems with time-delays

$$\begin{aligned} \dot{z}(t) &= A_0 z(t) + A_1 z(t-\tau) + Bu(t), \\ y(t) &= Cz(t) \end{aligned}$$

Port-Hamiltonian formulation for time-delay systems (wish list)

- Hamiltonian is explicitly included in the system dynamics
- pH systems vs. passivity

Roadmap:

- Formulate time-delay system as infinite-dimensional system (cf. Curtain/Zwart)
- Use operator KYP inequality to derive infinite-dimensional pH formulation
- Rewrite again as a time-delay system

Time-delay systems as infinite-dimensional system

$$
\begin{aligned}
\dot{z}(t) &= A_0 z(t) + A_1 z(t-\tau) + Bu(t), \\
y(t) &= Cz(t), \\
z(t) &= \phi(t) && \text{for } t \in [-\tau, 0]
\end{aligned}
$$

Time-delay systems as infinite-dimensional system

$$
\begin{aligned}
\dot{z}(t) &= A_0 z(t) + A_1 z(t-\tau) + Bu(t), \\
y(t) &= Cz(t), \\
z(t) &= \phi(t) && \text{for } t \in [-\tau, 0]
\end{aligned}
$$

Steps

- Appropriate Hilbert space: $\mathcal{X} := \mathbb{R}^N \times \mathcal{L}_2([-\tau,0];\mathbb{R}^N)$

Time-delay systems as infinite-dimensional system

$$\begin{aligned}\dot{z}(t) &= A_0 z(t) + A_1 z(t-\tau) + Bu(t),\\ y(t) &= Cz(t),\\ z(t) &= \phi(t) && \text{for } t \in [-\tau, 0]\end{aligned}$$

Steps

- Appropriate Hilbert space: $\mathcal{X} := \mathbb{R}^N \times \mathcal{L}_2([-\tau,0];\mathbb{R}^N)$
- Operators

$$\mathcal{A}\begin{bmatrix} z \\ \phi \end{bmatrix} = \begin{bmatrix} A_0 z + A_1\phi(-\tau) \\ \frac{\mathrm{d}}{\mathrm{d}s}\phi \end{bmatrix}, \qquad \mathcal{B} := \begin{bmatrix} B \\ 0 \end{bmatrix}, \qquad \mathcal{C} := \begin{bmatrix} C & 0 \end{bmatrix}$$

Time-delay systems as infinite-dimensional system

$$\begin{aligned}
\dot{z}(t) &= A_0 z(t) + A_1 z(t-\tau) + Bu(t), \\
y(t) &= Cz(t), \\
z(t) &= \phi(t) && \text{for } t \in [-\tau, 0]
\end{aligned}$$

Steps

- Appropriate Hilbert space: $\mathcal{X} := \mathbb{R}^N \times \mathcal{L}_2([-\tau,0];\mathbb{R}^N)$
- Operators

$$\mathcal{A}\begin{bmatrix} z \\ \phi \end{bmatrix} = \begin{bmatrix} A_0 z + A_1\phi(-\tau) \\ \frac{\mathrm{d}}{\mathrm{d}s}\phi \end{bmatrix}, \qquad \mathcal{B} := \begin{bmatrix} B \\ 0 \end{bmatrix}, \qquad \mathcal{C} := \begin{bmatrix} C & 0 \end{bmatrix}$$

$$D(\mathcal{A}) = \left\{ \begin{bmatrix} z \\ \phi \end{bmatrix} \in \mathcal{X} \;\middle|\; \begin{array}{l} \phi \text{ is absolutely continuous, } \frac{\mathrm{d}}{\mathrm{d}s}\phi \in \mathcal{L}_2([-\tau,0];\mathbb{R}^N), \\ \phi(0) = z \end{array} \right\}$$

Time-delay systems as infinite-dimensional system

$$\begin{aligned} \dot{z}(t) &= A_0 z(t) + A_1 z(t-\tau) + Bu(t), \\ y(t) &= Cz(t), \\ z(t) &= \phi(t) && \text{for } t \in [-\tau, 0] \end{aligned}$$

Equivalent operator formulation

$$\begin{aligned} \dot{x} &= \mathcal{A}x + \mathcal{B}u, \\ y &= \mathcal{C}x \end{aligned}$$

Operator KYP inequality

$$-\mathcal{W}(\mathcal{Q}) = \begin{bmatrix} \mathcal{A}^*\mathcal{Q} + \mathcal{Q}\mathcal{A} & \mathcal{Q}\mathcal{B} - \mathcal{C}^* \\ \mathcal{B}^*\mathcal{Q} - \mathcal{C} & 0 \end{bmatrix} \leq 0 \tag{1}$$

Operator KYP inequality

$$-\mathcal{W}(\mathcal{Q}) = \begin{bmatrix} \mathcal{A}^*\mathcal{Q} + \mathcal{Q}\mathcal{A} & \mathcal{Q}\mathcal{B} - \mathcal{C}^* \\ \mathcal{B}^*\mathcal{Q} - \mathcal{C} & 0 \end{bmatrix} \leq 0 \tag{1}$$

Adjoint operator

$$\mathcal{A}^* \begin{bmatrix} q \\ \psi \end{bmatrix} = \begin{bmatrix} A_0^\top q + \psi(0) \\ -\frac{\mathrm{d}}{\mathrm{d}s}(\psi - A_1^\top q \mathbb{1}_{[-\tau,0]}) \end{bmatrix}$$

$$D(\mathcal{A}^*) = \left\{ \begin{bmatrix} q \\ \psi \end{bmatrix} \in \mathcal{X} \;\middle|\; \begin{array}{l} \psi - A_1^\top q \mathbb{1}_{[-\tau,0]} \text{ is absolutely continuous,} \\ \frac{\mathrm{d}}{\mathrm{d}s}(z - A_1^\top q) \in L_2([-\tau;0];\mathbb{R}^N) \text{ and } z(-\tau) = A_1^\top q \end{array} \right\}.$$

Operator KYP inequality

$$-\mathcal{W}(\mathcal{Q}) = \begin{bmatrix} \mathcal{A}^*\mathcal{Q} + \mathcal{Q}\mathcal{A} & \mathcal{Q}\mathcal{B} - \mathcal{C}^* \\ \mathcal{B}^*\mathcal{Q} - \mathcal{C} & 0 \end{bmatrix} \leq 0 \tag{1}$$

Adjoint operator

$$\mathcal{A}^* \begin{bmatrix} q \\ \psi \end{bmatrix} = \begin{bmatrix} A_0^\top q + \psi(0) \\ -\frac{\mathrm{d}}{\mathrm{d}s}(\psi - A_1^\top q \mathbb{1}_{[-\tau,0]}) \end{bmatrix}$$

$$D(\mathcal{A}^*) = \left\{ \begin{bmatrix} q \\ \psi \end{bmatrix} \in \mathcal{X} \;\middle|\; \begin{array}{l} \psi - A_1^\top q \mathbb{1}_{[-\tau,0]} \text{ is absolutely continuous,} \\ \frac{\mathrm{d}}{\mathrm{d}s}(z - A_1^\top q) \in L_2([-\tau;0];\mathbb{R}^N) \text{ and } z(-\tau) = A_1^\top q \end{array} \right\}.$$

port-Hamiltonian system

$$\begin{aligned} \mathcal{Q}\dot{x} &= \mathcal{Q}\mathcal{A}x + \mathcal{Q}\mathcal{B}u \\ y &= \mathcal{C}x \end{aligned}$$

Operator KYP inequality

$$-\mathcal{W}(\mathcal{Q}) = \begin{bmatrix} \mathcal{A}^*\mathcal{Q} + \mathcal{Q}\mathcal{A} & \mathcal{Q}\mathcal{B} - \mathcal{C}^* \\ \mathcal{B}^*\mathcal{Q} - \mathcal{C} & 0 \end{bmatrix} \leq 0 \qquad (1)$$

Adjoint operator

$$\mathcal{A}^* \begin{bmatrix} q \\ \psi \end{bmatrix} = \begin{bmatrix} A_0^\top q + \psi(0) \\ -\frac{\mathrm{d}}{\mathrm{d}s}(\psi - A_1^\top q \mathbb{1}_{[-\tau,0]}) \end{bmatrix}$$

$$D(\mathcal{A}^*) = \left\{ \begin{bmatrix} q \\ \psi \end{bmatrix} \in \mathcal{X} \;\middle|\; \begin{array}{l} \psi - A_1^\top q \mathbb{1}_{[-\tau,0]} \text{ is absolutely continuous,} \\ \frac{\mathrm{d}}{\mathrm{d}s}(z - A_1^\top q) \in L_2([-\tau;0];\mathbb{R}^N) \text{ and } z(-\tau) = A_1^\top q \end{array} \right\}.$$

port-Hamiltonian system

$$\begin{aligned} \mathcal{Q}\dot{x} &= (\mathcal{J} - \mathcal{R})x + \mathcal{Q}\mathcal{B}u \\ y &= \mathcal{C}x \end{aligned}$$

Assumptions on the Hamiltonian

$$\mathcal{Q} = \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 \\ \mathcal{Q}_2^* & \mathcal{Q}_3 \end{bmatrix}, \qquad \mathcal{H}(z,\phi) = \frac{1}{2}\left\langle \begin{bmatrix} z \\ \phi \end{bmatrix}, \mathcal{Q}\begin{bmatrix} z \\ \phi \end{bmatrix} \right\rangle = \frac{1}{2} z^\top \mathcal{Q}_1 z + z^\top \mathcal{Q}_2 \phi + \frac{1}{2}\int_{-\tau}^{0} \phi(s)^\top (\mathcal{Q}_3\phi)(s)\mathrm{d}s$$

Assumptions on the Hamiltonian

$$\mathcal{Q} = \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 \\ \mathcal{Q}_2^* & \mathcal{Q}_3 \end{bmatrix}, \qquad \mathcal{H}(z,\phi) = \tfrac{1}{2}\left\langle \begin{bmatrix} z \\ \phi \end{bmatrix}, \mathcal{Q}\begin{bmatrix} z \\ \phi \end{bmatrix} \right\rangle = \tfrac{1}{2} z^\top \mathcal{Q}_1 z + z^\top \mathcal{Q}_2 \phi + \tfrac{1}{2}\int_{-\tau}^{0} \phi(s)^\top (\mathcal{Q}_3 \phi)(s)\mathrm{d}s$$

Standard delay theory: Lyapunov-Krasovskii functional

$$\mathcal{H}(z,\phi) = \tfrac{1}{2} z^\top Q_1 z + \int_{-\tau}^{0} \phi(s)^\top Q_3 \phi(s)\mathrm{d}s$$

Assumptions on the Hamiltonian

$$\mathcal{Q} = \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 \\ \mathcal{Q}_2^* & \mathcal{Q}_3 \end{bmatrix}, \qquad \mathcal{H}(z,\phi) = \tfrac{1}{2}\left\langle \begin{bmatrix} z \\ \phi \end{bmatrix}, \mathcal{Q}\begin{bmatrix} z \\ \phi \end{bmatrix} \right\rangle = \tfrac{1}{2} z^\top \mathcal{Q}_1 z + z^\top \mathcal{Q}_2 \phi + \tfrac{1}{2}\int_{-\tau}^{0} \phi(s)^\top (\mathcal{Q}_3\phi)(s)\mathrm{d}s$$

Standard delay theory: Lyapunov-Krasovskii functional

$$\mathcal{H}(z,\phi) = \tfrac{1}{2} z^\top Q_1 z + \int_{-\tau}^{0} \phi(s)^\top Q_3 \phi(s)\mathrm{d}s$$

Crucial assumptions $\mathcal{Q}_2 = 0$, $\mathcal{Q}_3 = Q_3 \in \mathbb{R}^{N\times N}$

Assumptions on the Hamiltonian

$$\mathcal{Q} = \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 \\ \mathcal{Q}_2^* & \mathcal{Q}_3 \end{bmatrix}, \qquad \mathcal{H}(z,\phi) = \tfrac{1}{2}\left\langle \begin{bmatrix} z \\ \phi \end{bmatrix}, \mathcal{Q}\begin{bmatrix} z \\ \phi \end{bmatrix} \right\rangle = \tfrac{1}{2} z^\top \mathcal{Q}_1 z + z^\top \mathcal{Q}_2 \phi + \tfrac{1}{2}\int_{-\tau}^{0} \phi(s)^\top (\mathcal{Q}_3\phi)(s)\mathrm{d}s$$

Standard delay theory: Lyapunov-Krasovskii functional

$$\mathcal{H}(z,\phi) = \tfrac{1}{2} z^\top Q_1 z + \int_{-\tau}^{0} \phi(s)^\top Q_3 \phi(s)\mathrm{d}s$$

Crucial assumptions $\mathcal{Q}_2 = 0$, $\mathcal{Q}_3 = Q_3 \in \mathbb{R}^{N\times N}$

$$\mathcal{J}\begin{bmatrix} z \\ \phi \end{bmatrix} = -\tfrac{1}{2}\begin{bmatrix} A_0^\top Q_1 z + Q_3 z - Q_1 A_0 z - Q_1\phi(-\tau) \\ -\tfrac{\mathrm{d}}{\mathrm{d}s}(2Q_3\phi - A_1^\top Q_1 x \mathbb{1}_{[-\tau,0]}) \end{bmatrix},$$

$$\mathcal{R}\begin{bmatrix} z \\ \phi \end{bmatrix} = -\tfrac{1}{2}\begin{bmatrix} A_0^\top Q_1 z + Q_3 z + Q_1 A_0 z + Q_1\phi(-\tau) \\ -\tfrac{\mathrm{d}}{\mathrm{d}s}(2Q_3\phi - A_1^\top Q_1 x \mathbb{1}_{[-\tau,0]}) \end{bmatrix}$$

Port-Hamiltonian systems with time-delays

Definition

A time-delay system of the form

$$\begin{aligned} H_1\dot{z}(t) &= (J-R)z(t) - A_1 z(t-\tau) + Bu(t), \\ y(t) &= B^\top z(t) \end{aligned}$$

with Hamiltonian

$$\mathcal{H}(z\big|_{[t-\tau,t]}) = \tfrac{1}{2} z(t)^\top H_1 z(t) + \int_{t-\tau}^{t} z(s)^\top H_2 z(s)\,\mathrm{d}s$$

is called a port-Hamiltonian delay system, if $H_1 > 0$, $H_2 \geq 0$, $J = -J^\top$ and

$$\begin{bmatrix} z(t) \\ z(t-\tau) \end{bmatrix}^\top \begin{bmatrix} R-H_2 & A_1 \\ A_1^\top & H_2 \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-\tau) \end{bmatrix} \geq 0$$

along any solution z.

Some remarks

$$
\begin{aligned}
H_1\dot{z}(t) &= (J-R)z(t) - A_1 z(t-\tau) + Bu(t),\\
y(t) &= B^\top z(t)
\end{aligned}
$$

Lemma

A port-Hamiltonian delay system satisfies the dissipation inequality.

Some remarks

$$\begin{aligned} H_1\dot{z}(t) &= (J-R)z(t) - A_1 z(t-\tau) + Bu(t), \\ y(t) &= B^\top z(t) \end{aligned}$$

(Sufficient) Dissipation condition

$$\begin{bmatrix} z(t) \\ z(t-\tau) \end{bmatrix}^\top \begin{bmatrix} R-H_2 & A_1 \\ A_1^\top & H_2 \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-\tau) \end{bmatrix} \geq 0$$

Special cases

- $A_1 = 0 \rightsquigarrow$ we recover classical pH systems (by setting $H_2 = 0$)
- $\tau = 0 \rightsquigarrow$ we recover classical pH systems (by setting $H_2 = 0$)

Some remarks

$$
\begin{aligned}
H_1\dot{z}(t) &= (J-R)z(t) - A_1 z(t-\tau) + Bu(t),\\
y(t) &= B^\top z(t)
\end{aligned}
$$

(Sufficient) Dissipation condition

$$
\begin{bmatrix} z(t) \\ z(t-\tau) \end{bmatrix}^\top \begin{bmatrix} R-H_2 & A_1 \\ A_1^\top & H_2 \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-\tau) \end{bmatrix} \geq 0
$$

Special cases

- $A_1 = 0 \rightsquigarrow$ we recover classical pH systems (by setting $H_2 = 0$)
- $\tau = 0 \rightsquigarrow$ we recover classical pH systems (by setting $H_2 = 0$)
- $H_2 > 0 \rightsquigarrow$ we recover condition from the literature ([Niculescu, Lozano, 2001])

Example

$$\begin{aligned} H_1\dot{z}(t) &= (J-R)z(t) - A_1 z(t-\tau) + Bu(t), \\ y(t) &= B^\top z(t) \end{aligned}$$

Example

$$\begin{aligned} \dot{z}(t) &= -\alpha z(t) - \beta z(t-\tau) + u(t), \\ y(t) &= z(t) \end{aligned}$$

Example

$$\begin{aligned} H_1\dot{z}(t) &= (J-R)z(t) - A_1 z(t-\tau) + Bu(t), \\ y(t) &= B^\top z(t) \end{aligned}$$

Example

$$\begin{aligned} \dot{z}(t) &= -\alpha z(t) - \beta z(t-\tau) + u(t), \\ y(t) &= z(t) \end{aligned}$$

Set $H_1 = 1$, $R = \alpha$, $A_1 = \beta$, $B = 1$

Consequences

- $R \geq 0 \implies \alpha \geq 0$
- Find $\eta \geq 0$ such that

$$\begin{bmatrix} \alpha-\eta & \beta \\ \beta & \eta \end{bmatrix} \geq 0$$

Example

$$
\begin{aligned}
H_1 \dot{z}(t) &= (J - R)z(t) - A_1 z(t-\tau) + Bu(t), \\
y(t) &= B^\top z(t)
\end{aligned}
$$

Example

$$
\begin{aligned}
\dot{z}(t) &= -\alpha z(t) - \beta z(t-\tau) + u(t), \\
y(t) &= z(t)
\end{aligned}
$$

Set $H_1 = 1$, $R = \alpha$, $A_1 = \beta$, $B = 1$

Consequences

- $R \geq 0 \implies \alpha \geq 0$
- Find $\eta \geq 0$ such that

$$
\begin{bmatrix} \alpha - \eta & \beta \\ \beta & \eta \end{bmatrix} \geq 0 \iff \text{necessary \& sufficient condition for passivity}
$$

First implications

(Sufficient) Dissipation condition

$$\begin{bmatrix} z(t) \\ z(t-\tau) \end{bmatrix}^\top \begin{bmatrix} R - H_2 & A_1 \\ A_1^\top & H_2 \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-\tau) \end{bmatrix} \geq 0$$

First implications

(Sufficient) Dissipation condition

$$\begin{bmatrix} R - H_2 & A_1 \\ A_1^\top & H_2 \end{bmatrix} \geq 0$$

First implications

(Sufficient) Dissipation condition

$$\begin{bmatrix} R - S & Z \\ Z^\top & S \end{bmatrix} \geq 0$$

First implications

(Sufficient) Dissipation condition

$$\begin{bmatrix} R - S & Z \\ Z^\top & S \end{bmatrix} \geq 0$$

$$\left[\begin{array}{cc|cc} R_1 - S_1 & -S_2 & Z_1 & Z_2 \\ -S_2^\top & -S_3 & Z_3 & Z_4 \\ \hline Z_1^\top & Z_3^\top & S_1 & S_2 \\ Z_2^\top & Z_4^\top & S_2^\top & S_3 \end{array}\right]$$

First implications

(Sufficient) Dissipation condition

$$\begin{bmatrix} R - S & Z \\ Z^\top & S \end{bmatrix} \geq 0$$

$$\left[\begin{array}{cc|cc} R_1 - S_1 & -S_2 & Z_1 & Z_2 \\ -S_2^\top & 0 & Z_3 & Z_4 \\ \hline Z_1^\top & Z_3^\top & S_1 & S_2 \\ Z_2^\top & Z_4^\top & S_2^\top & 0 \end{array}\right]$$

First implications

(Sufficient) Dissipation condition

$$\begin{bmatrix} R - S & Z \\ Z^\top & S \end{bmatrix} \geq 0$$

$$\left[\begin{array}{cc|cc} R_1 - S_1 & 0 & Z_1 & Z_2 \\ 0 & 0 & Z_3 & Z_4 \\ \hline Z_1^\top & Z_3^\top & S_1 & 0 \\ Z_2^\top & Z_4^\top & 0 & 0 \end{array}\right]$$

First implications

(Sufficient) Dissipation condition

$$\begin{bmatrix} R - S & Z \\ Z^\top & S \end{bmatrix} \geq 0$$

$$\left[\begin{array}{cc|cc} R_1 - S_1 & 0 & Z_1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline Z_1^\top & 0 & S_1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

First implications

(Sufficient) Dissipation condition

$$\begin{bmatrix} R - S & Z \\ Z^\top & S \end{bmatrix} \geq 0$$

$$\left[\begin{array}{cc|cc} R_1 - S_1 & 0 & Z_1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline Z_1^\top & 0 & S_1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rightsquigarrow \operatorname{Ker}(R) \subseteq \operatorname{Ker}(Z)$

First implications

(Sufficient) Dissipation condition

$$\begin{bmatrix} R - S & Z \\ Z^\top & S \end{bmatrix} \geq 0$$

$$\left[\begin{array}{cc|cc} R_1 - S_1 & 0 & Z_1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline Z_1^\top & 0 & S_1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rightsquigarrow \operatorname{Ker}(R) \subseteq \operatorname{Ker}(Z)$

Open questions

- When does suitable S (previously H_2) exists?
- How to construct such an S?

Summary and challenges

Time-delay pH systems

- Rewrite time-delay system as infinite dimensional system
- Obtain pH formulation via operator KYP (assuming a special solution)
- Rewrite as time-delay system to obtain pH formulation for time-delay systems

Summary and challenges

Time-delay pH systems

- Rewrite time-delay system as infinite dimensional system
- Obtain pH formulation via operator KYP (assuming a special solution)
- Rewrite as time-delay system to obtain pH formulation for time-delay systems

Many open questions

- $S\left(H_2\right)$ not included in the system dynamics
- Only special solutions of the operator KYP $\rightsquigarrow$ time-dependent kernels
- What happens if delay is induced by delayed interconnection?
- Construction for actual application

Universität Stuttgart

B. Unger
Independent Junior Research Group for Dynamical Systems
SC SimTech, Universität Stuttgart

eMail benjamin.unger@simtech.uni-stuttgart.de
Telefon +49-711-685 60114