ETH zürich

Transverse feedback passivation in control of power networks

Catalin Arghir

ETH & Beyond Gravity

Trends on dissipativity in systems and control workshop

Part I
Problem setup

Methodology overview

Passivity-based-feedback as guideline for control design (choice of output plays a major role)

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Consider the synchronisation example:

$$\begin{aligned}
\dot{\theta} &= \omega \\
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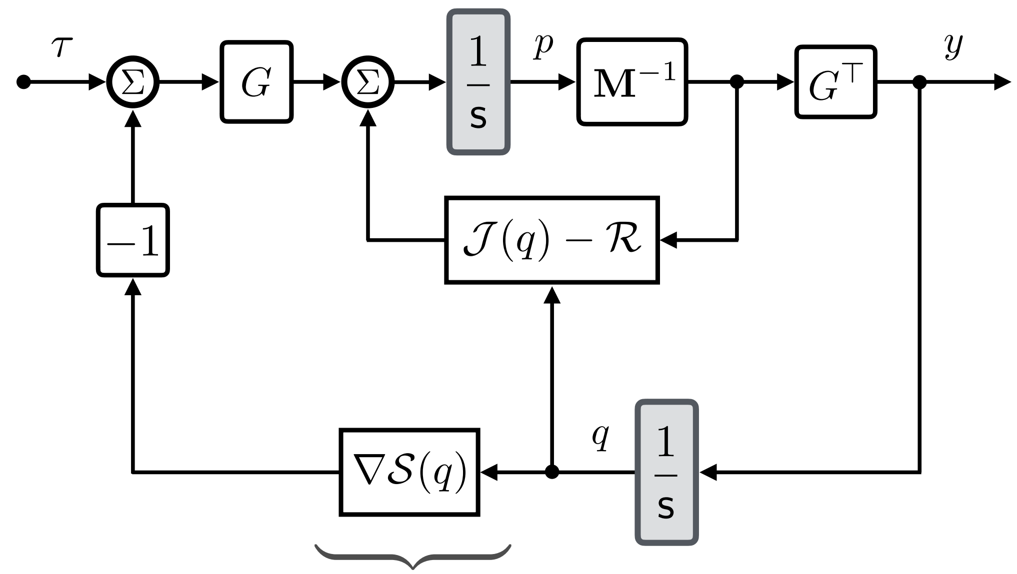

- potential energy t.b.d.
- under-actuated
- constant mass-matrix

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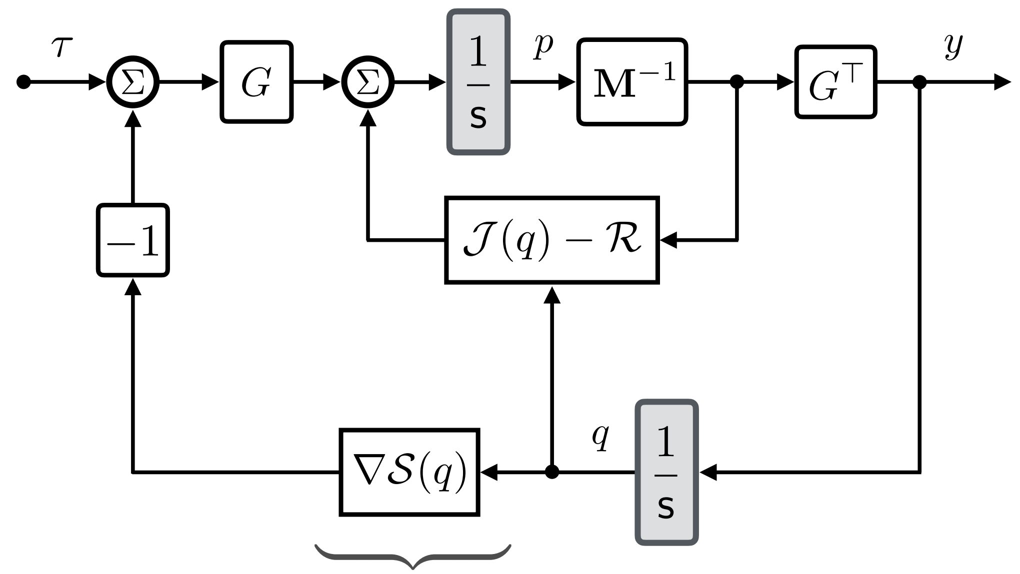
assume S = 0

lossless energy exchange

Laplacian + damping



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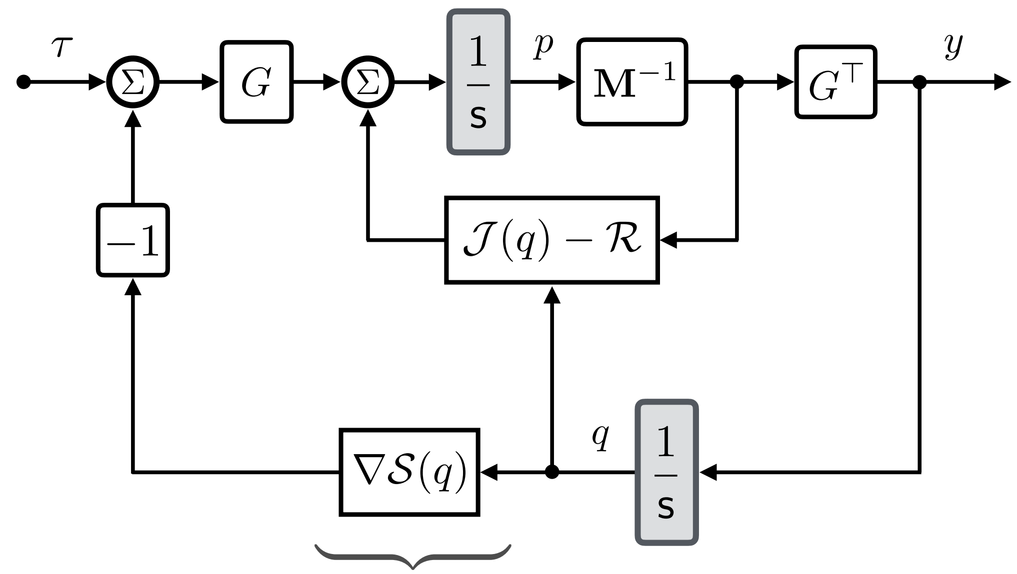

Naive PBF: $\tau = -Ky$ and zero-state detectability

would mean: $x(t) \to 0 \text{ as } t \to \infty$

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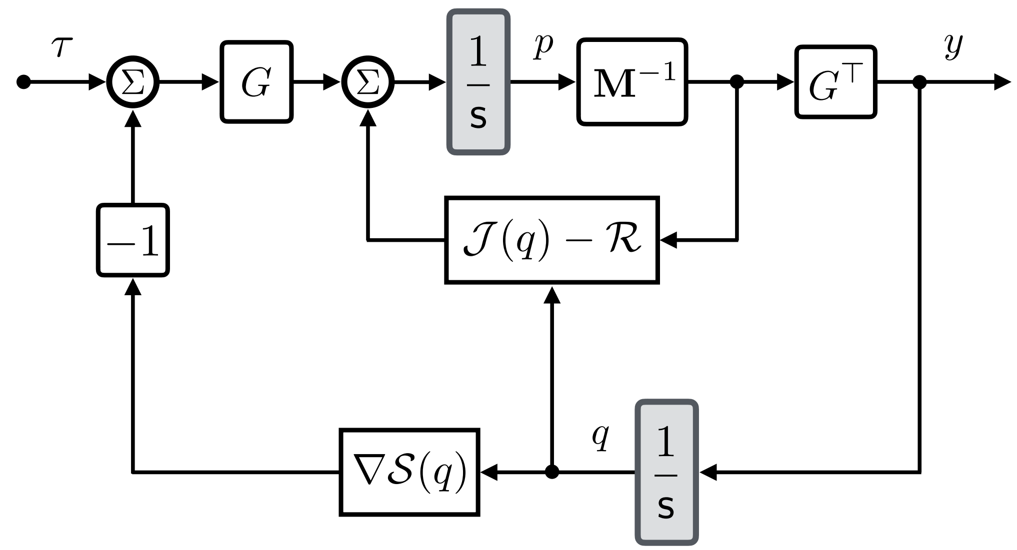

Suppose we choose PBF for: $\tilde{y} = \omega - \omega_0 \mathbb{1}$

the zero dynamics become a cascade

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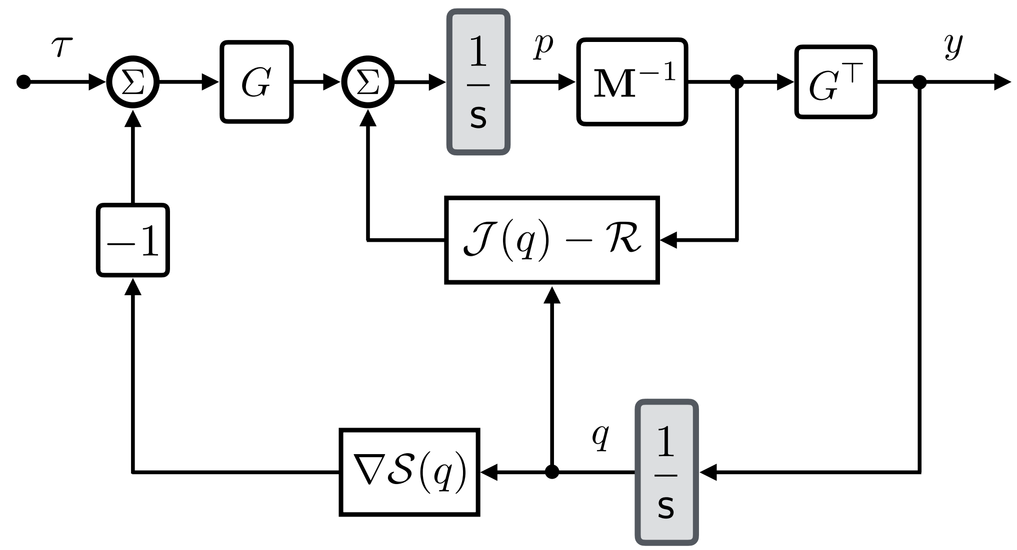

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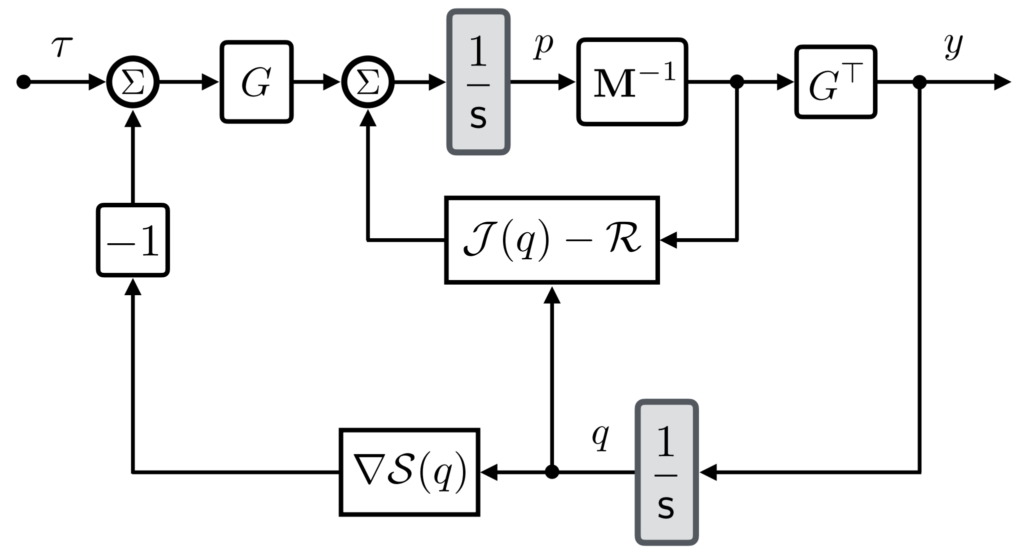

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\quad \equiv \quad
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\dot{\xi} &= \boldsymbol{j}\omega_0\xi \\
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assuming non-resonance,

admits $\mathcal{SSL} = \{(\xi, z) : z = \Pi\xi\}$

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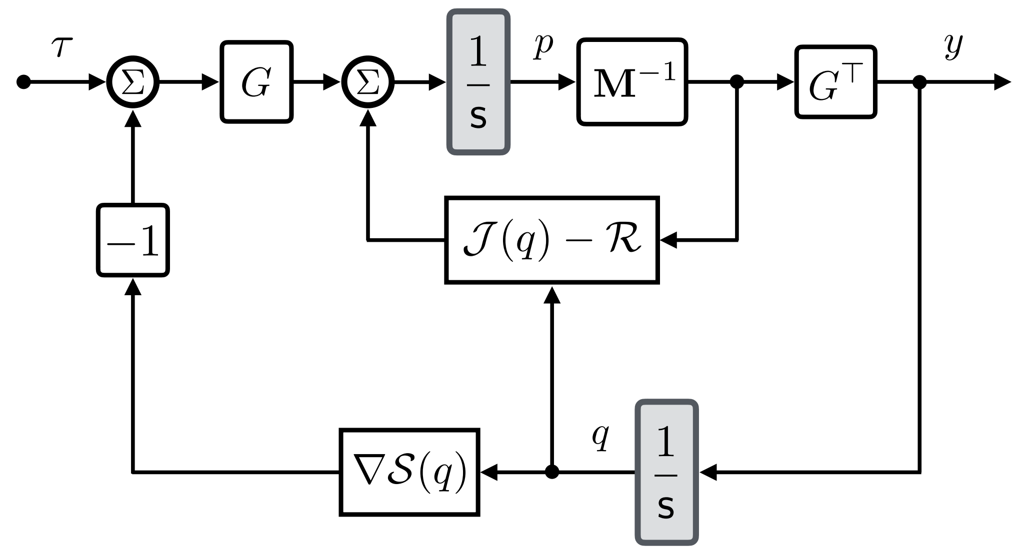

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Refine output: $\bar{y} = \begin{bmatrix} \omega - \omega_0\mathbb{1} \\ z - \pi(\theta) \end{bmatrix}$

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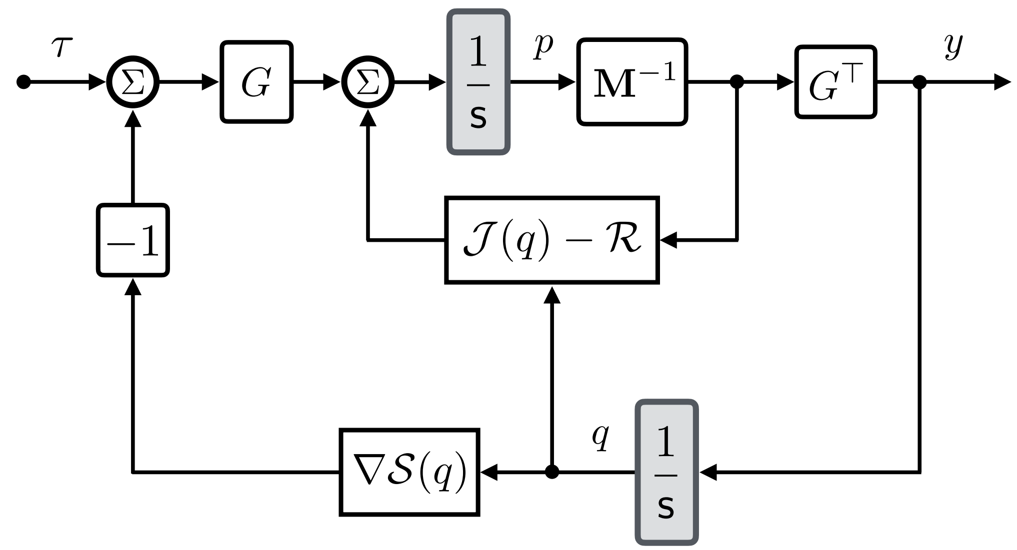

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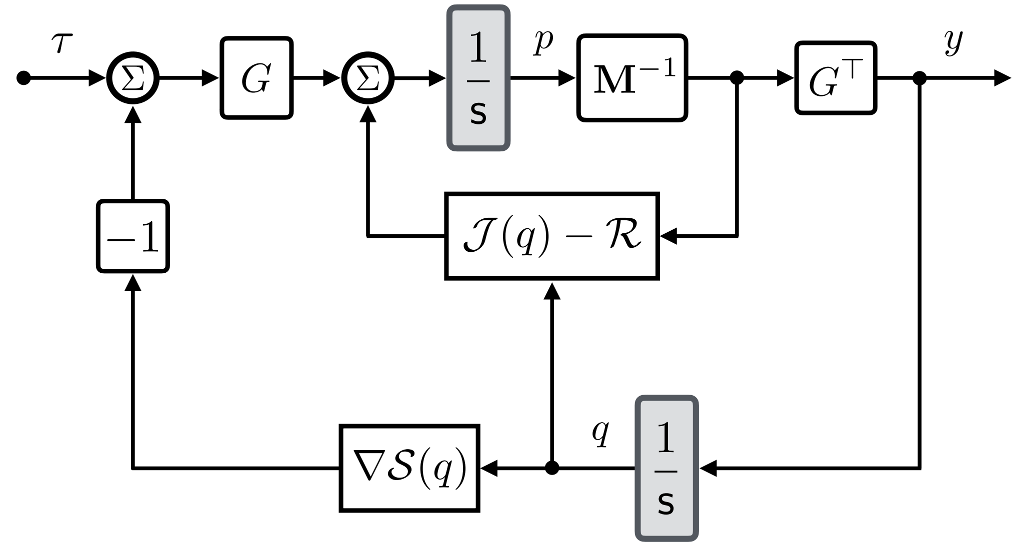

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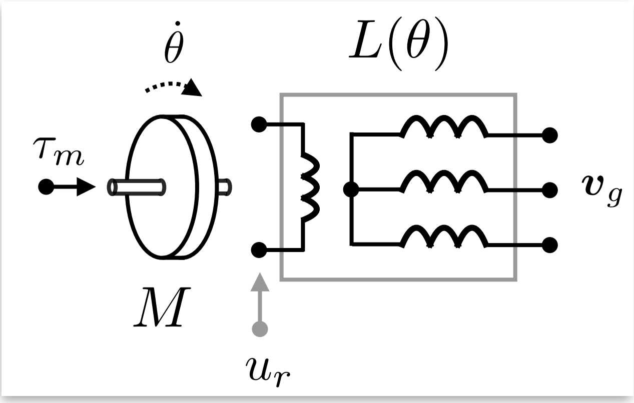
→ *Passive error dynamics:* $\tau = \underbrace{D\omega_0\mathbb{1} - g(\theta)^\top\pi(\theta)}_{\text{controlled invariance}} + \underbrace{\frac{\partial}{\partial\theta}\left(\frac{1}{2}\tilde{z}^\top T\tilde{z}\right)}_{\text{passivation}}$, where $\tilde{z} = z - \pi(\theta)$

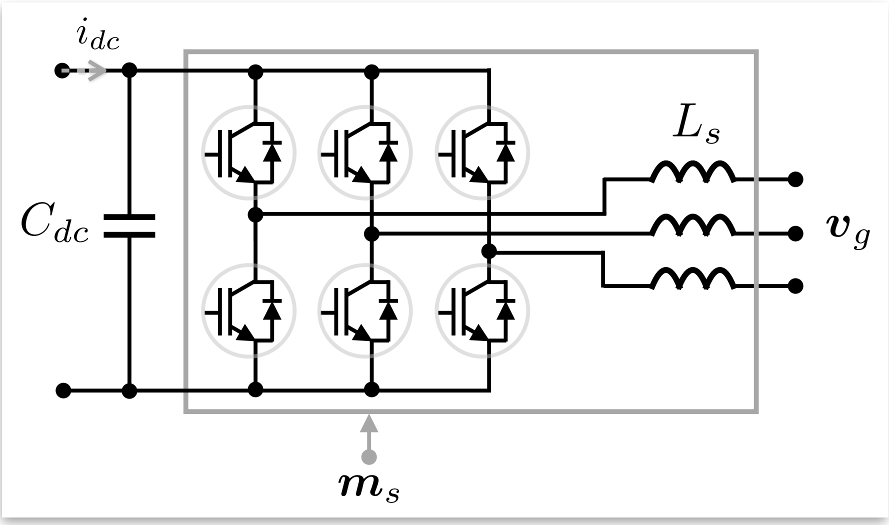
Part II
Feedback equivalence step

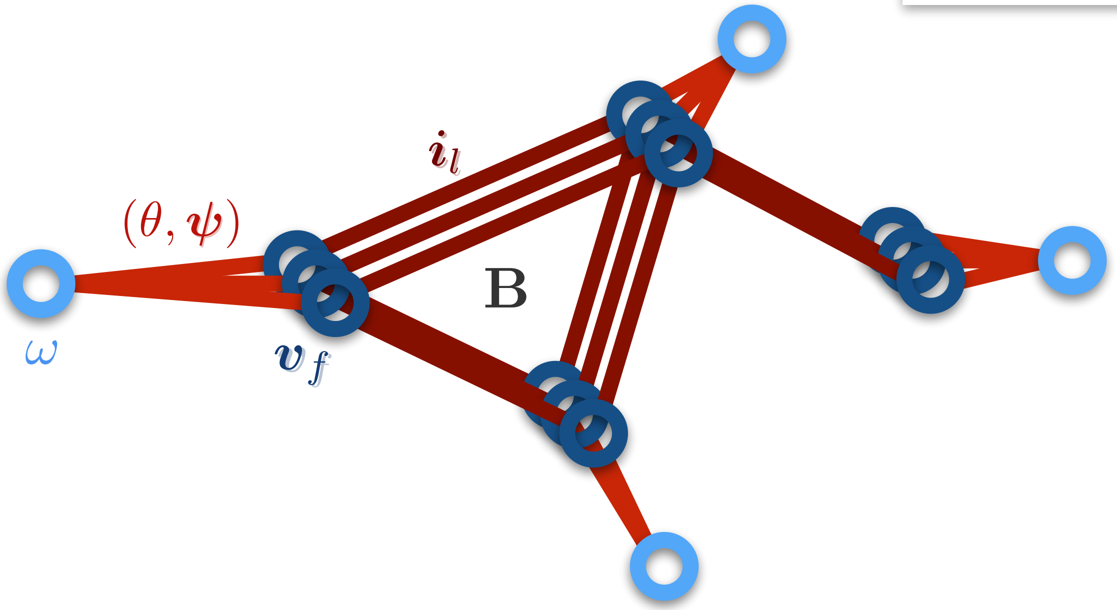
Problem overview

Consider a network of synchronous machines (and DC/AC converters)

- Q: how to actuate both in a unified manner
- Q: how to define & achieve hierarchical specifications







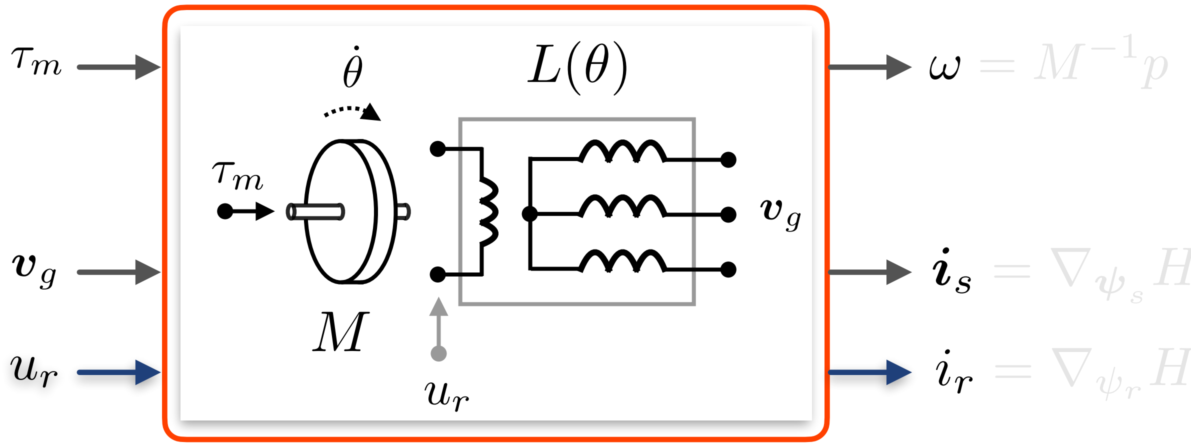
Problem overview

$$L(\theta) = \begin{bmatrix} l_s & 0 & l_m \cos\theta \\ 0 & l_s & l_m \sin\theta \\ l_m \cos\theta & l_m \sin\theta & l_r \end{bmatrix}$$

Consider the synchronous machine

- passive with storage

$$H = \frac{1}{2} M^{-1} p^2 + \frac{1}{2} \begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix}^\top L(\theta)^{-1} \begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix}$$



generalized coordinate: θ

$\mathcal{J}$ independent on x

$$\dot{x} = (\mathcal{J} - \mathcal{R})\nabla H + Gu$$
$$y = G^\top \nabla H$$

Main obstructions

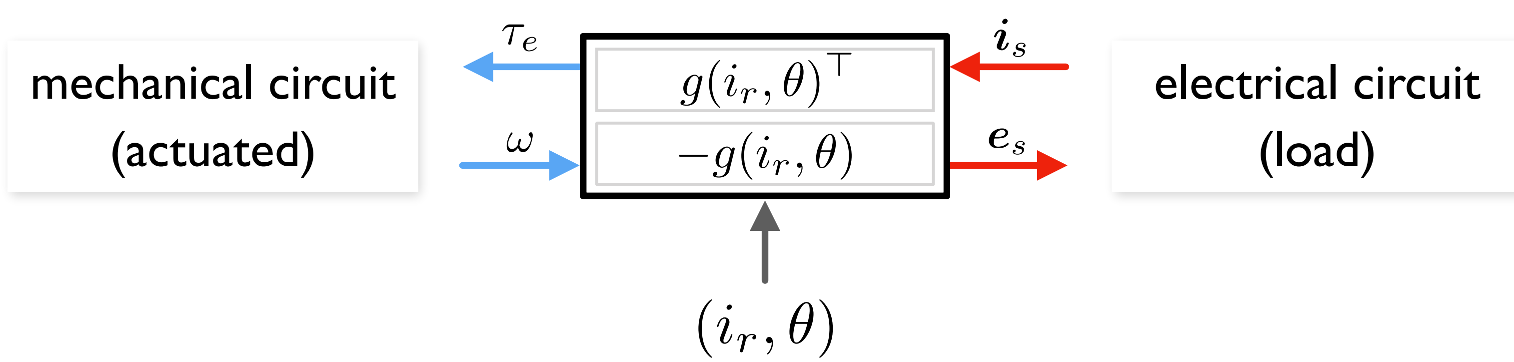
- circle nonlinearity
- modulated energy conversion element

$$\mathrm{g}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

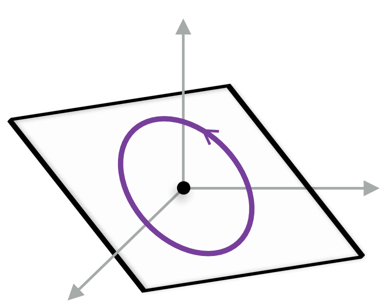
EMF $\longrightarrow$ $\boldsymbol{e}_s = l_m i_r \boldsymbol{R}_\theta \mathrm{g}_2 \omega$

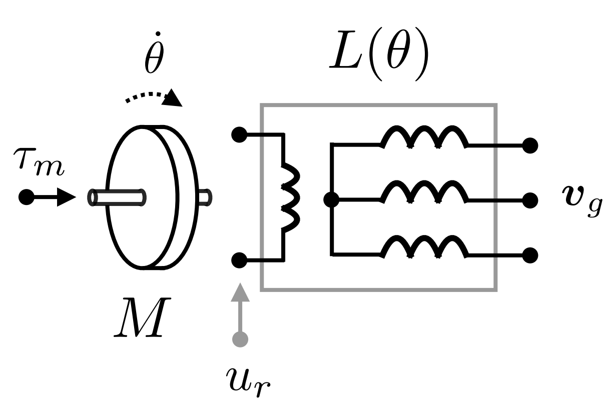
electrical torque $\longrightarrow$ $\tau_e = -l_m i_r \mathrm{g}_2^\top \boldsymbol{R}_\theta^\top \boldsymbol{i}_s$

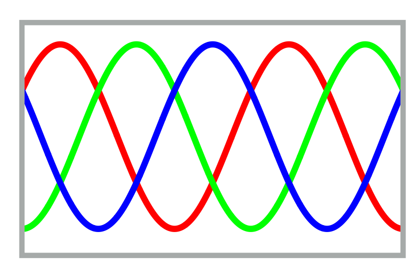
$\omega,\ \tau_e \in \mathbb{R}$



$\boldsymbol{i}_s,\ \boldsymbol{e}_s \in \mathbb{R}^2$



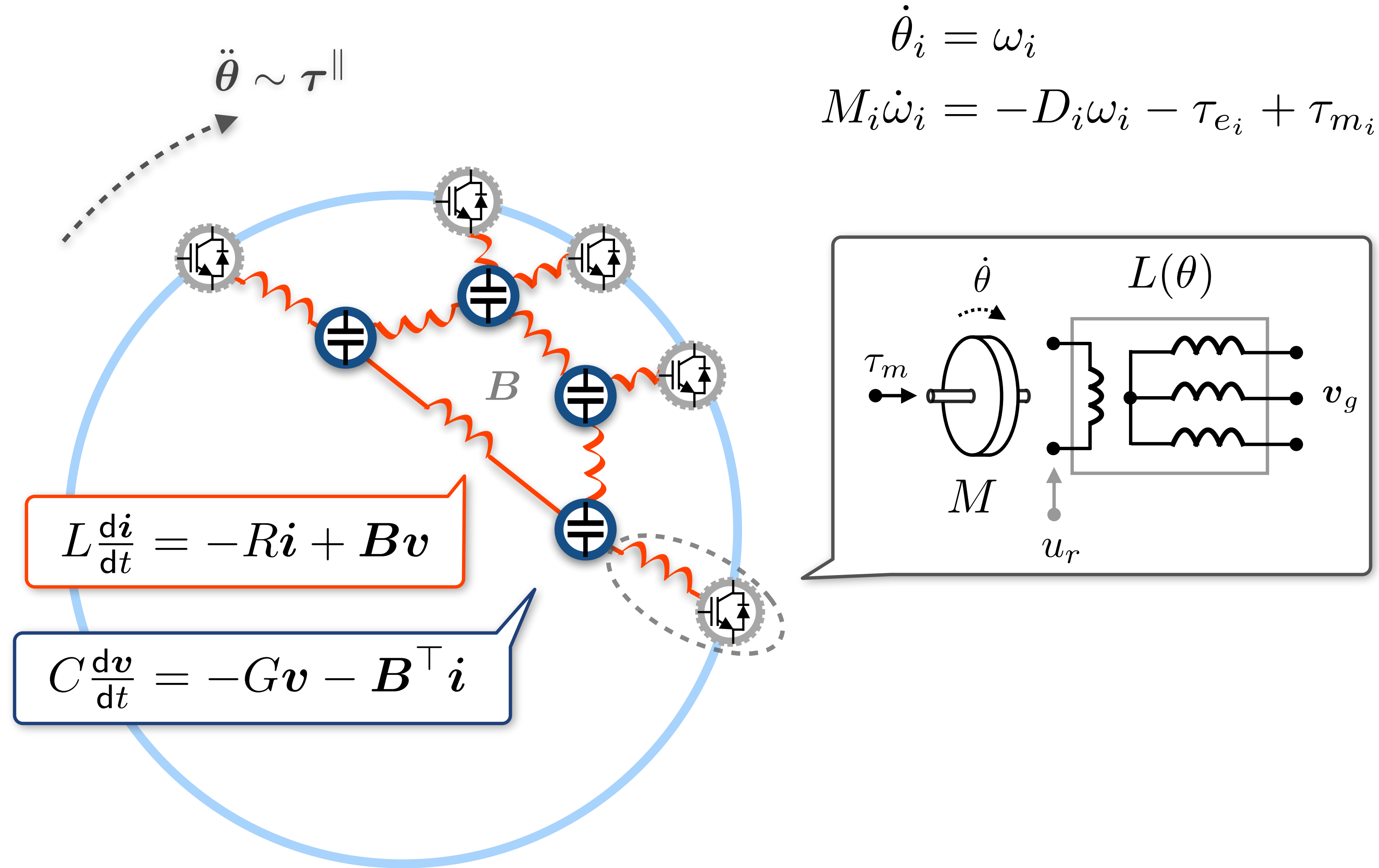




Multi-machine model: amplitude coupling

- first-principles
- dynamic stator
- dynamic lines
- multiple equilibria

- objective: angle and amplitude regulation



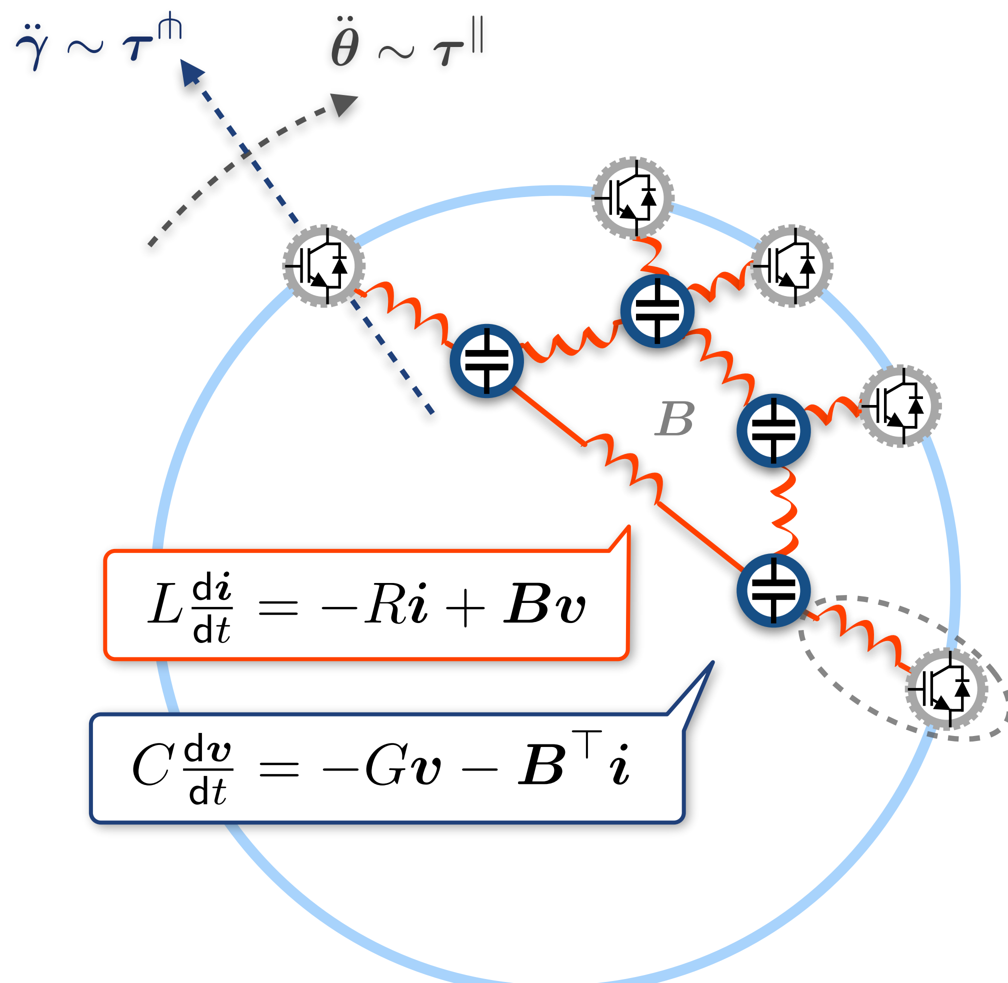
= synchronous machine or inverter

= three-phase inductance

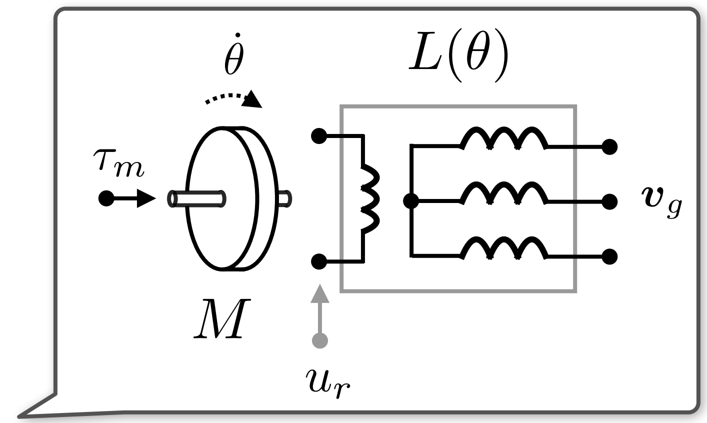
= AC-bus capacitance

Multi-machine model: how to deal with excitation?

- first-principles
- dynamic stator
- dynamic lines
- multiple equilibria



$$\dot{\theta}_i = \omega_i$$
$$M_i\dot{\omega}_i = -D_i\omega_i - \tau_{e_i} + \tau_{m_i}$$



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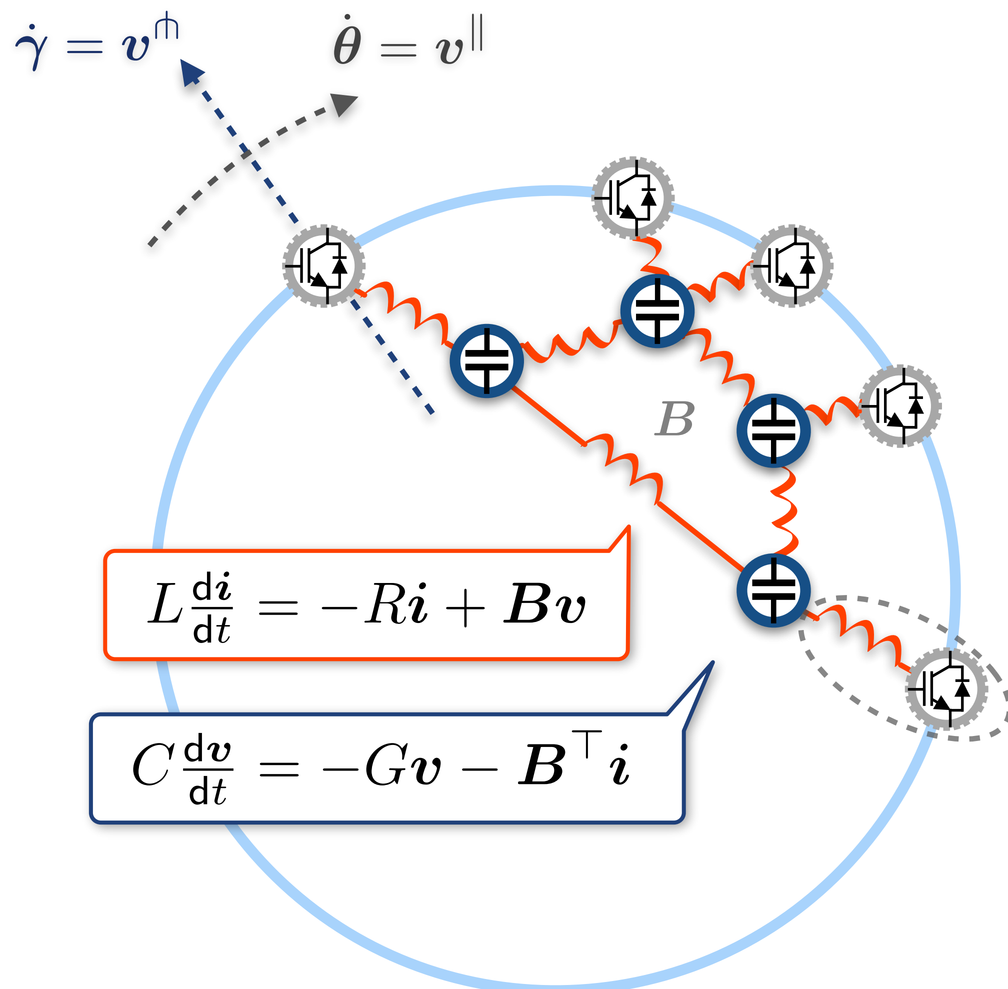
Multi-machine model: augment amplitude coupling

Consider the quotient space $\mathbb{R}_{>0} = \mathbb{R}^2_* / \mathbb{S}^1$

- define $\gamma_r = \ln i_r$
- assign $\dot{\gamma}_r = \nu$



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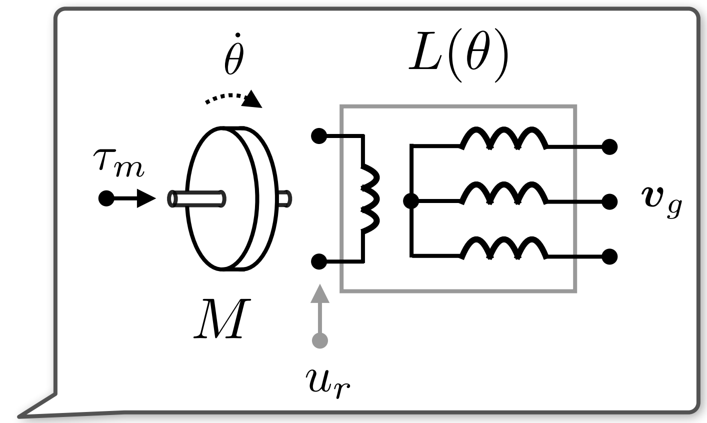

- objective: angle and log(amplt.) regulation

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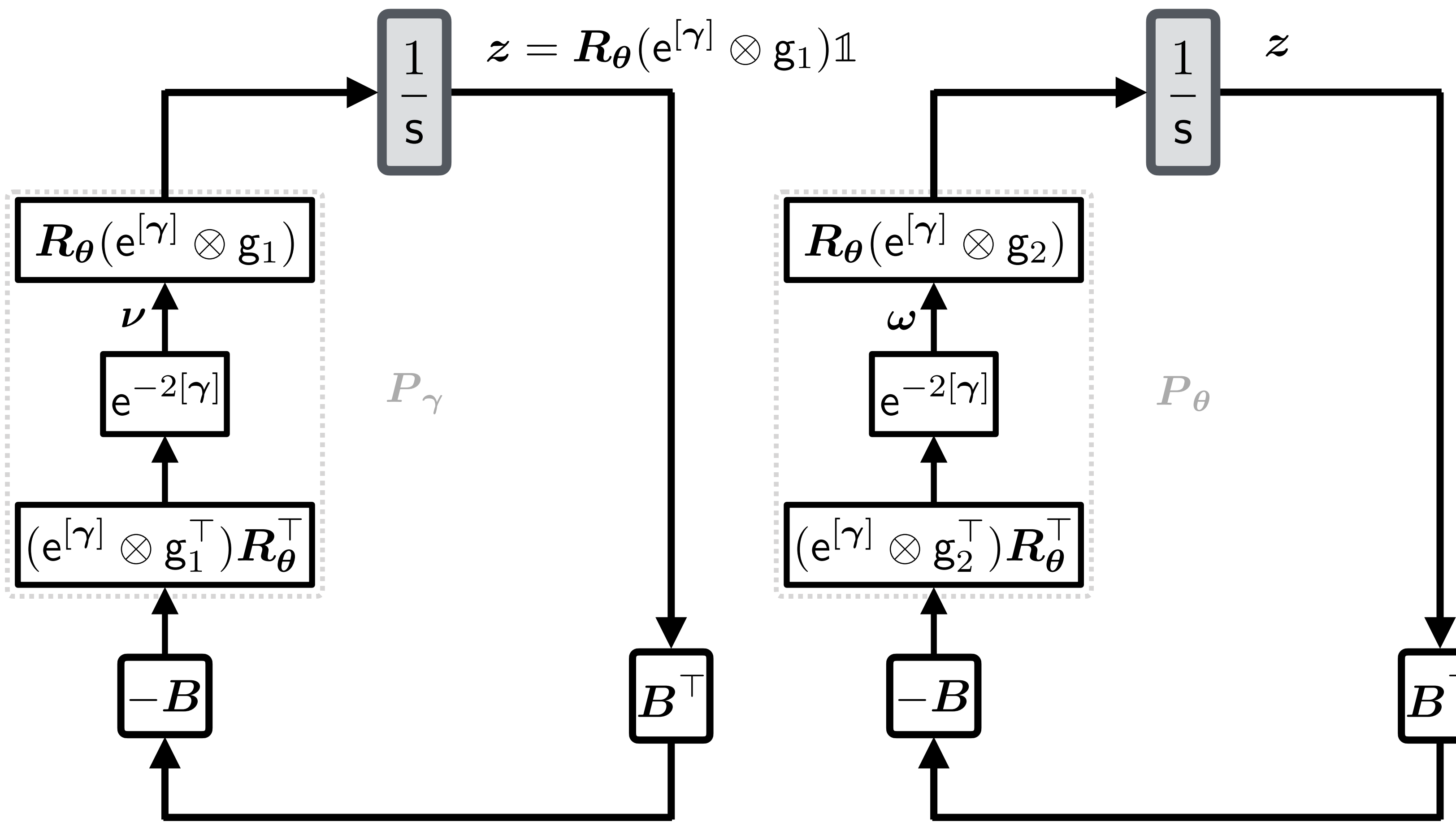
Ray-circle complementarity

Euclidean embedding $z = \exp(\gamma + i\theta)$ best reflects active and reactive power decomposition

- observed in the electromotive force expression

(Faraday's law) $$\boldsymbol{e}_s = l_m i_r \boldsymbol{R}_\theta \left(\frac{1}{i_r}\frac{\mathrm{d}i_r}{\mathrm{d}t}\mathrm{g}_1 + \frac{\mathrm{d}\theta}{\mathrm{d}t}\mathrm{g}_2 \right)$$ $\mathrm{g}_1 = \begin{bmatrix}1\\0\end{bmatrix}, \mathrm{g}_2 = \begin{bmatrix}0\\1\end{bmatrix}$



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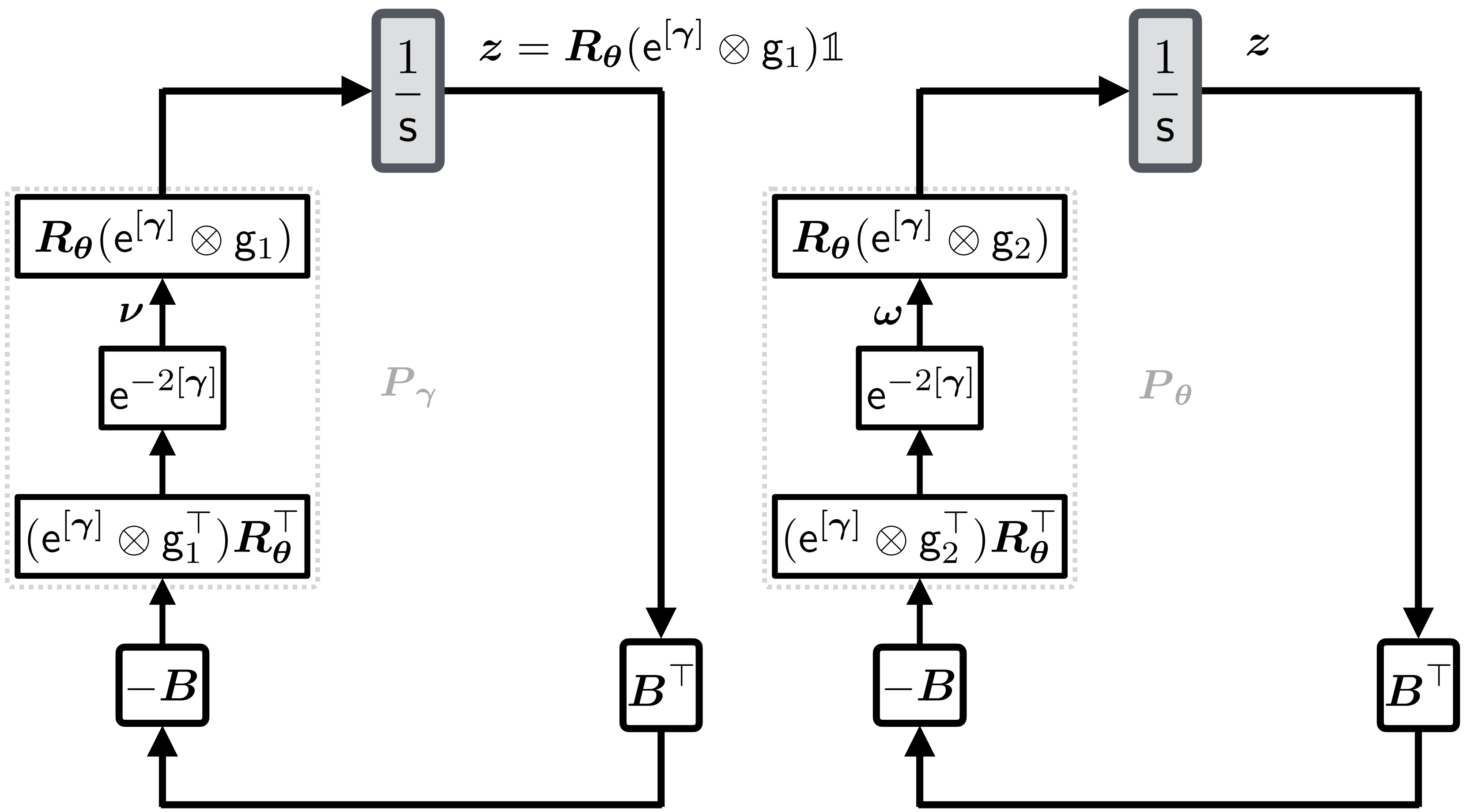

$$\frac{\mathrm{d}}{\mathrm{d}t} i_{r_i} = -\sum_{j\in\mathcal{N}_i}(i_{r_i} - i_{r_j}) \qquad \frac{\mathrm{d}}{\mathrm{d}t}\theta_i = -\sum_{j\in\mathcal{N}_i}\sin(\theta_i - \theta_j)$$

$$i_{r_i} = \mathrm{e}^{\gamma_i} = \|z_i\| \qquad \theta_i = \mathrm{atan2}(z_{2_i}, z_{1_i})$$

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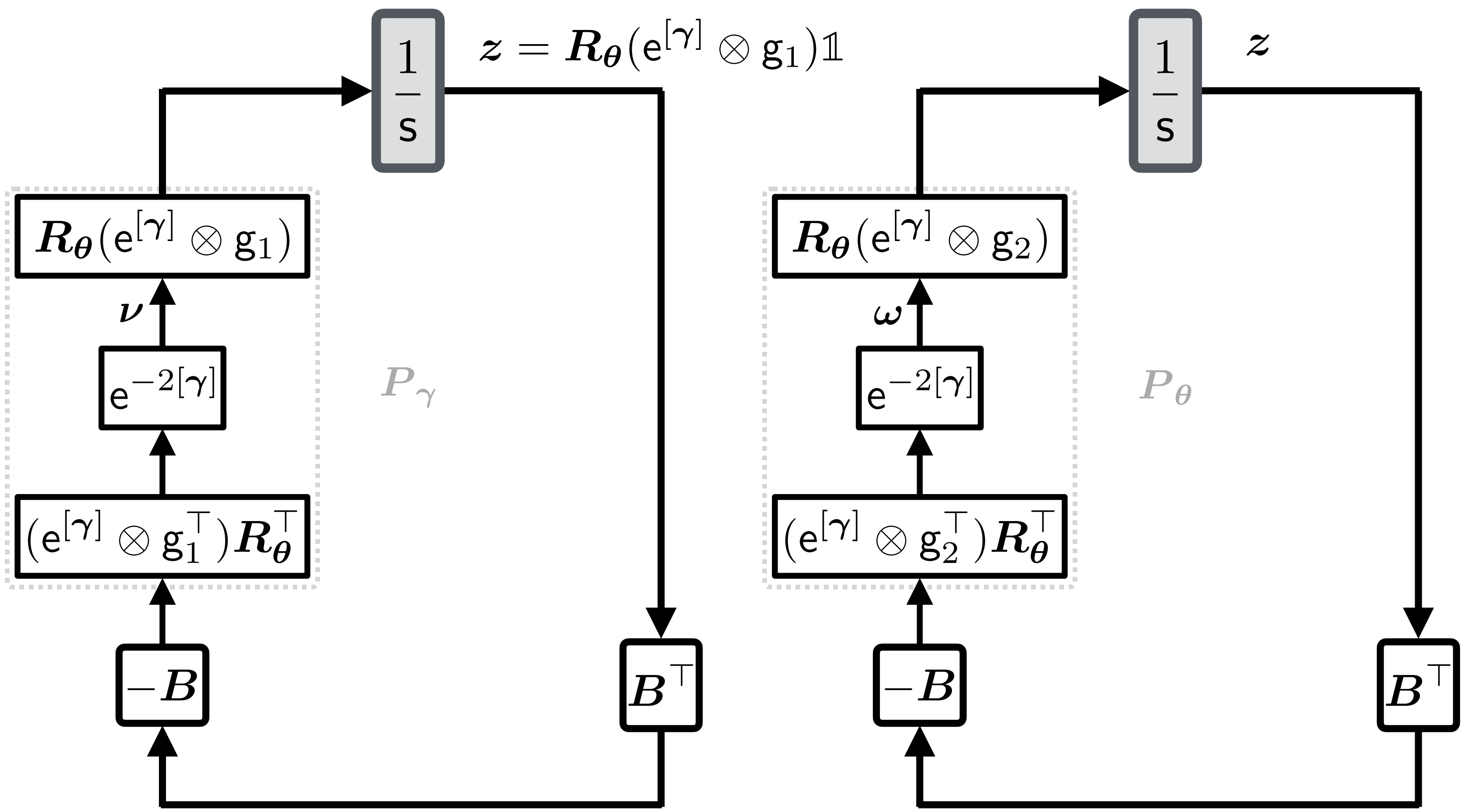

$$\frac{\mathrm{d}}{\mathrm{d}t}\gamma_i = \sum_{j \in \mathcal{N}_i} \left(\mathrm{e}^{(\gamma_j - \gamma_i)} - 1 \right) \qquad \frac{\mathrm{d}}{\mathrm{d}t}\theta_i = \sum_{j \in \mathcal{N}_i} \sin(\theta_j - \theta_i)$$

$$\gamma_i = \ln \|z_i\| \qquad \theta_i = \mathrm{atan2}(z_{2_i}, z_{1_i})$$

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Consider the synchronous machine

- passive with storage

$$H = \tfrac{1}{2} M^{-1} p^2 + \tfrac{1}{2} \begin{bmatrix} \boldsymbol{\psi}_s \\ \psi_r \end{bmatrix}^\top L(\theta)^{-1} \begin{bmatrix} \boldsymbol{\psi}_s \\ \psi_r \end{bmatrix}$$

$$\tau_m \rightarrow \quad \dot p = -D\omega - \tau_e + \tau_m \quad \rightarrow \omega = M^{-1} p$$

$$\dot\theta = \omega$$

$$\boldsymbol{v}_g \rightarrow \quad \dot{\boldsymbol{\psi}}_s = -R_s \boldsymbol{i}_s + \boldsymbol{v}_g \quad \rightarrow \boldsymbol{i}_s = \nabla_{\boldsymbol{\psi}_s} H$$

$$u_r \rightarrow \quad \dot\psi_r = -r_r i_r + u_r \quad \rightarrow i_r = \nabla_{\psi_r} H$$

generalized coordinate: θ

$\mathcal{J}$ independent on x

$$\dot x = (\mathcal{J} - \mathcal{R}) \nabla H + Gu$$

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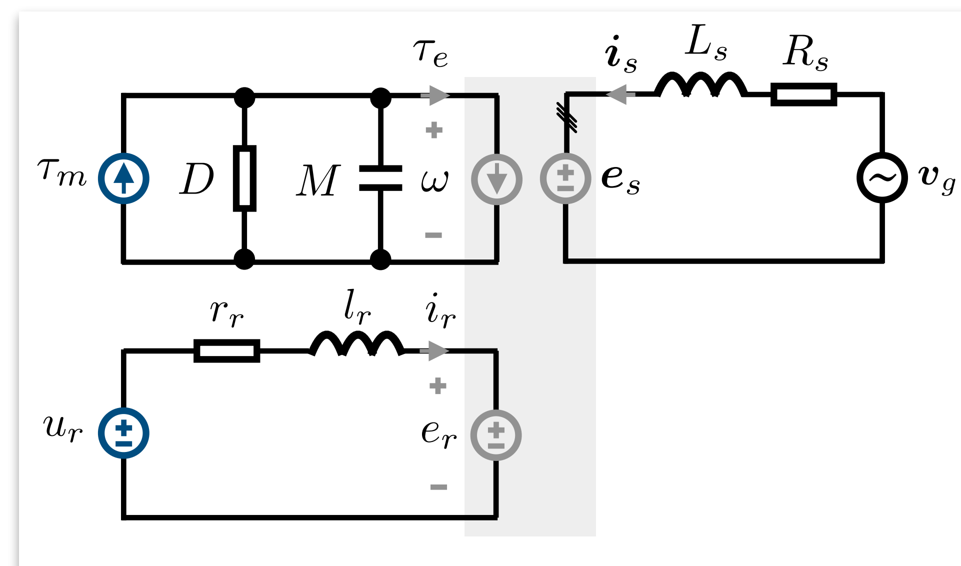
$\dot\theta = \omega$



Ray-circle matching

Consider the synchronous machine

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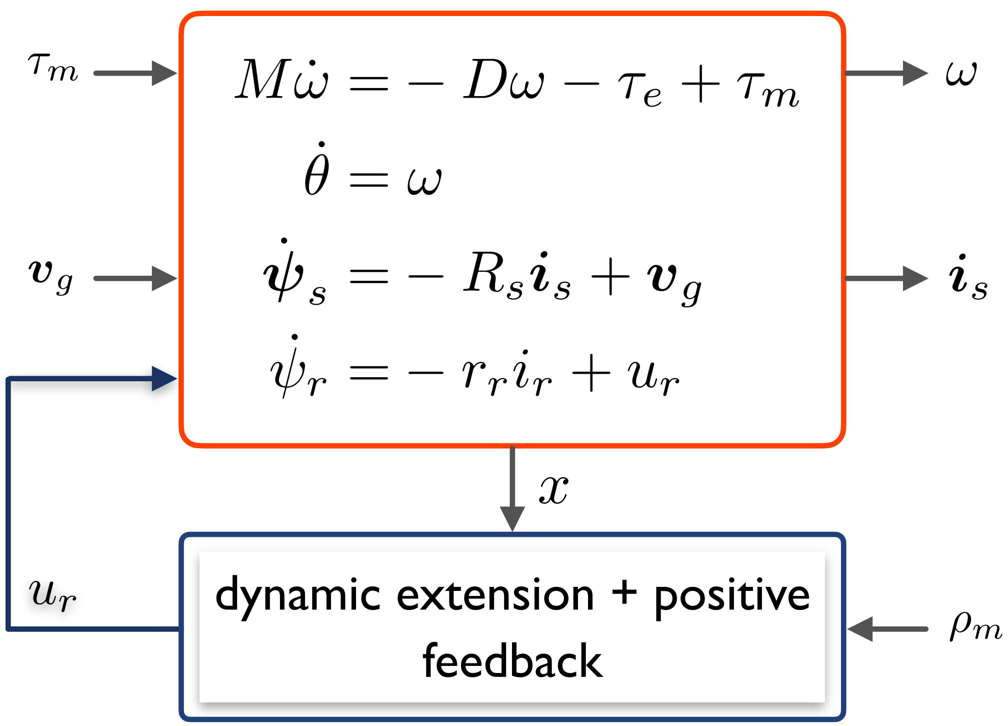

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$$\gamma_r = \ln i_r$$

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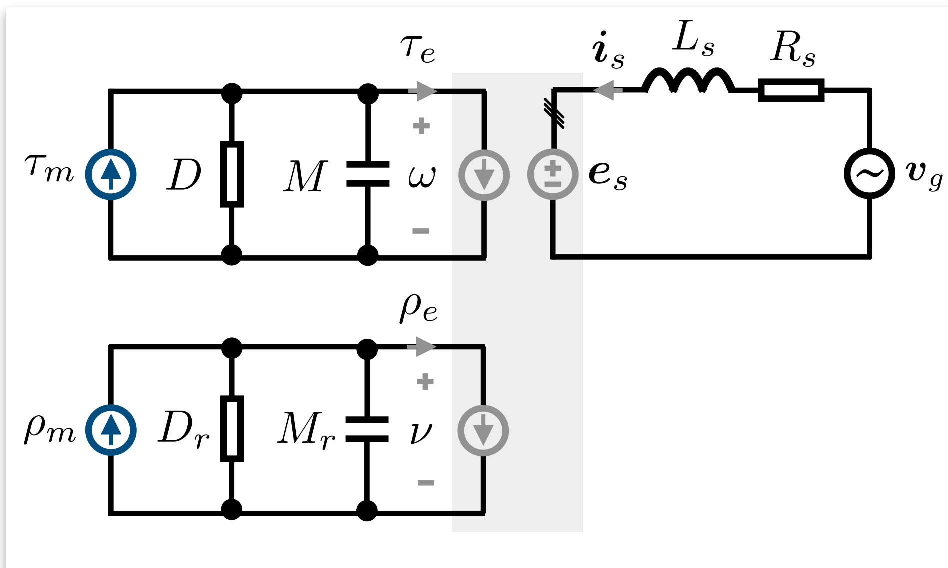
$$H = \tfrac{1}{2}M_r\nu^2 + \tfrac{1}{2}M\omega^2 + \tfrac{1}{2}\boldsymbol{i}_s^\top L_s\boldsymbol{i}_s$$



generalized coordinates: (θ, γ_r)

Mass matrix independent on $q = (\theta, \gamma_r)$

$$\dot{x} = (\mathcal{J}(q) - \mathcal{R})\nabla H + Gu$$
$$y = G^\top \nabla H$$

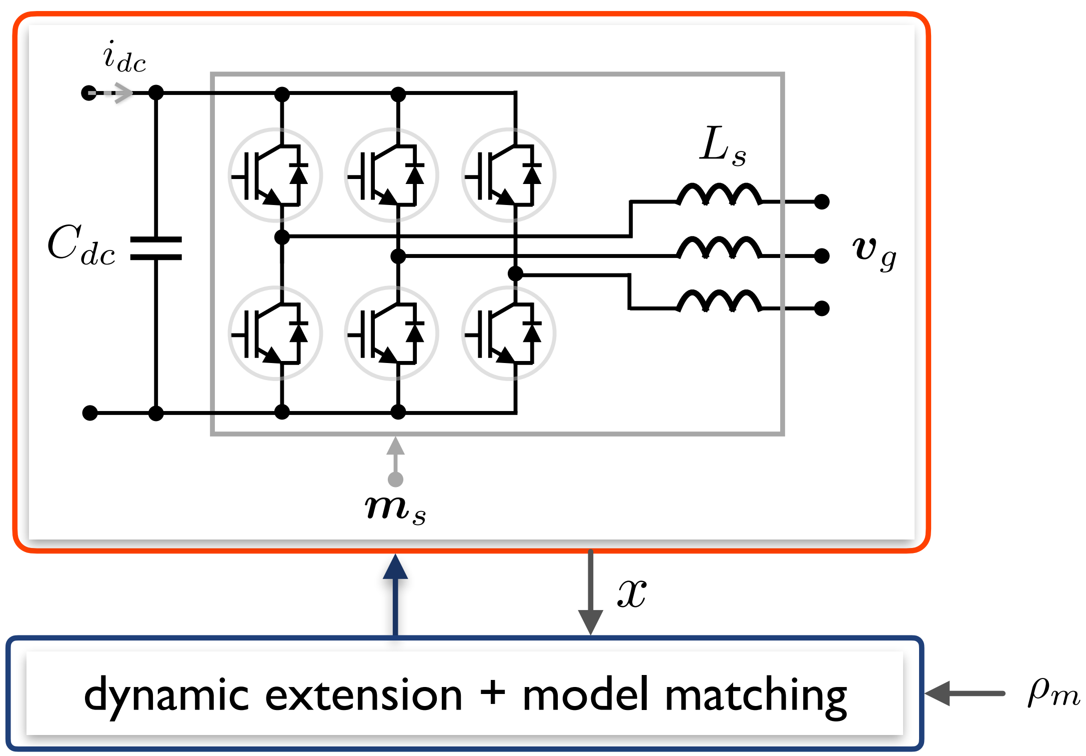
a feedback equivalence problem

Ray-circle matching

Consider the three-phase inverter

- passive with storage

$$H = \tfrac{1}{2} C_{dc} v_{dc}^2 + \tfrac{1}{2} \boldsymbol{i}_s^\top L_s \boldsymbol{i}_s$$



$$\dot{x} = (\mathcal{J}(m) - \mathcal{R})\nabla H + Gu$$
$$y = G^\top \nabla H$$

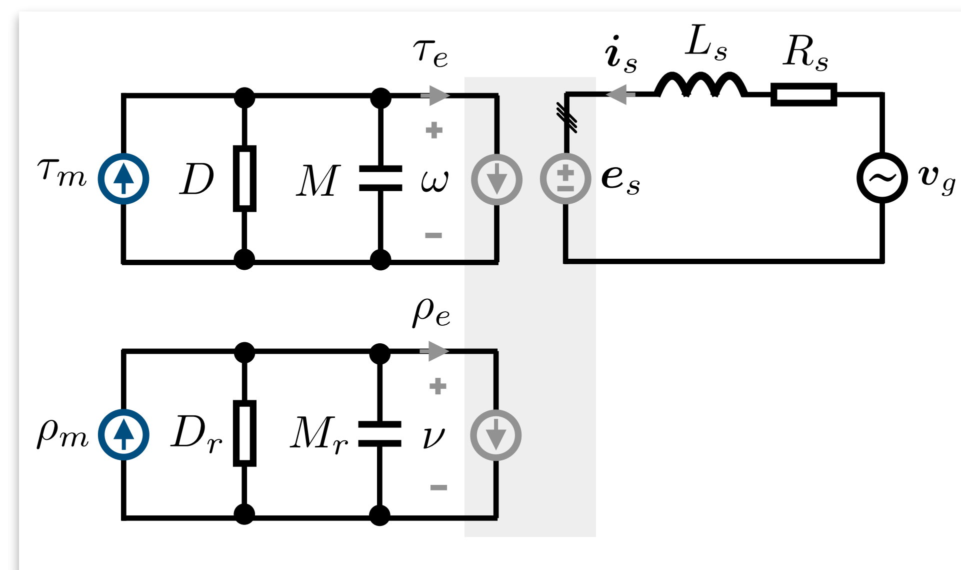
$\omega = \eta v_{dc}$

$\dot{\theta} = \omega$

$\dot{\gamma}_r = \nu$

- passive with storage

$$H = \tfrac{1}{2} M_r \nu^2 + \tfrac{1}{2} M \omega^2 + \tfrac{1}{2} \boldsymbol{i}_s^\top L_s \boldsymbol{i}_s$$



generalized coordinates: (θ, γ_r)

Mass matrix independent on $q = (\theta, \gamma_r)$

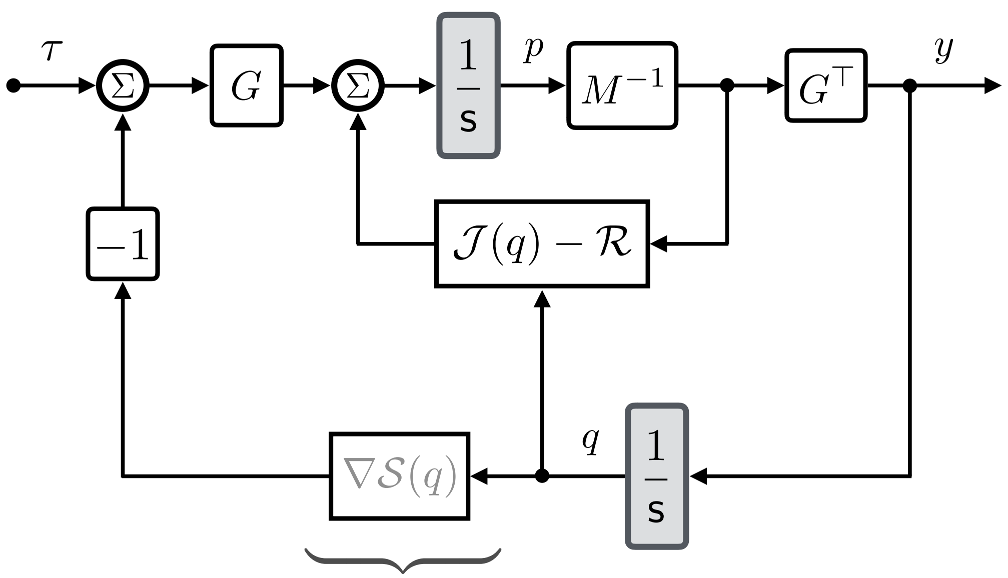
$$\dot{x} = (\mathcal{J}(q) - \mathcal{R})\nabla H + Gu$$
$$y = G^\top \nabla H$$

a feedback equivalence problem

Ray-circle matching

Target system:

- under-actuated
- constant mass-matrix

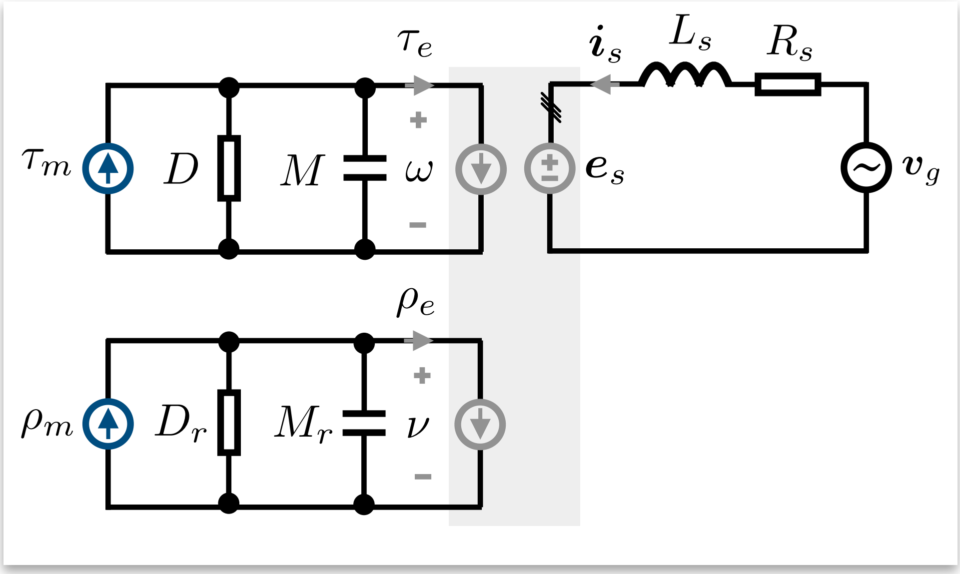


$\equiv$

- potential energy t.b.d.

- passive with storage

$$H = \tfrac{1}{2}M_r\nu^2 + \tfrac{1}{2}M\omega^2 + \tfrac{1}{2}\boldsymbol{i}_s^\top L_s \boldsymbol{i}_s$$



generalized coordinates: (θ, γ_r)

Mass matrix independent on $q = (\theta, \gamma_r)$

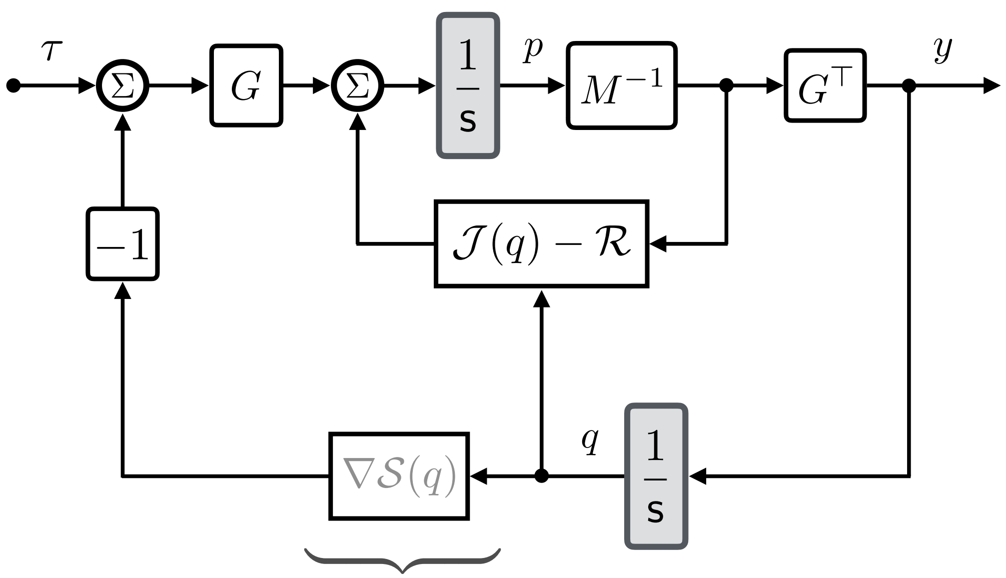
$$\dot{x} = (\mathcal{J}(q) - \mathcal{R})\nabla H + Gu$$
$$y = G^\top \nabla H$$

a feedback equivalence problem

Cone-torus matching

Networked system:

- passive with storage $\mathcal{H} = \frac{1}{2}\boldsymbol{\nu}^\top \boldsymbol{M_r}\boldsymbol{\nu} + \frac{1}{2}\boldsymbol{\omega}^\top \boldsymbol{M_a}\boldsymbol{\omega} + \frac{1}{2}\boldsymbol{i}_s^\top \boldsymbol{L_s}\boldsymbol{i}_s + \frac{1}{2}\boldsymbol{v}_f^\top \boldsymbol{C_f}\boldsymbol{v}_f + \frac{1}{2}\boldsymbol{i}_l^\top \boldsymbol{L_l}\boldsymbol{i}_l$



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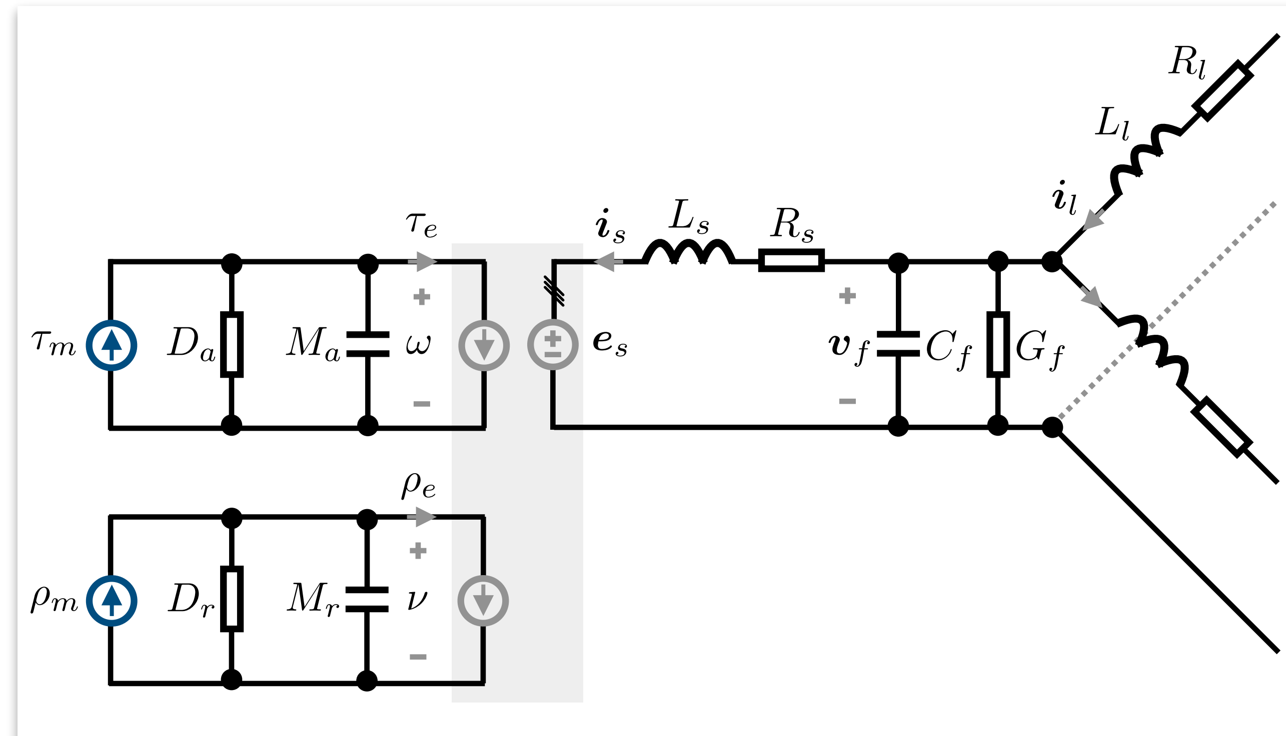


- potential energy t.b.d.

generalized coordinates: $(\boldsymbol{\theta}, \gamma_r)$

Mass matrix independent on $\boldsymbol{q} = (\boldsymbol{\theta}, \gamma_r)$

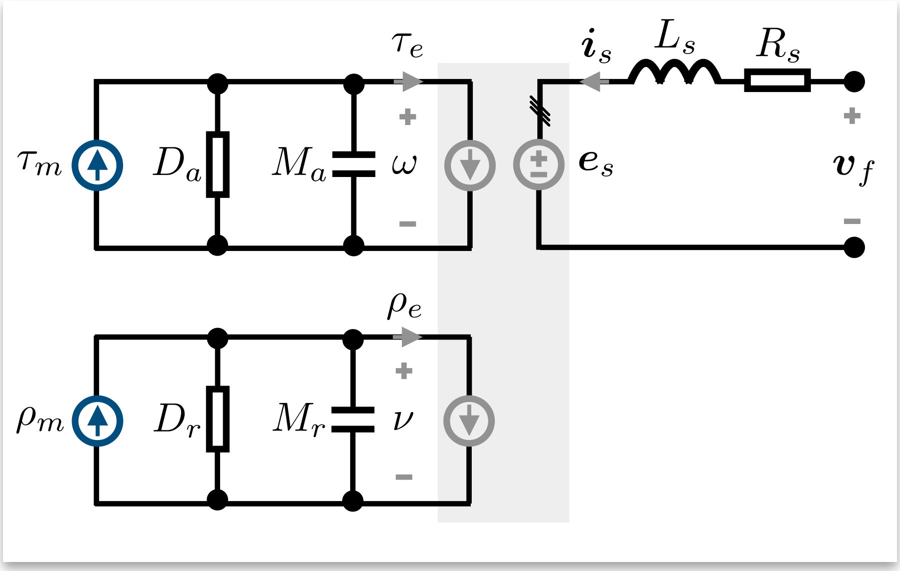
$$\dot{\boldsymbol{x}} = (J(\boldsymbol{q}) - R)\nabla\mathcal{H} + G\boldsymbol{u}$$

$$\boldsymbol{y} = G^\top \nabla\mathcal{H}$$

Part III
Solution setup

General strategy

- initial specs: energy bias (lose passivity)
- recover transverse passivity
- design potential energy function (tangential goal)
- stabilize the refined target set



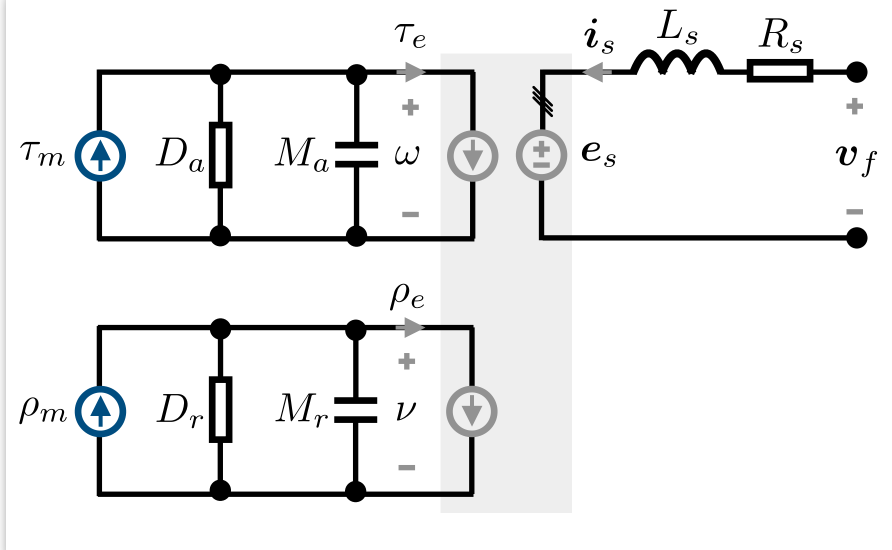
$$\begin{bmatrix} \boldsymbol{\rho}_m \\ \boldsymbol{\tau}_m \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ D_{\boldsymbol{a}}\omega_0 \mathbb{1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\rho}_e - \nabla_{\gamma_r} S \\ \boldsymbol{\tau}_e - \nabla_\theta S \end{bmatrix}}_{\text{controlled-invariance}} \underbrace{- \begin{bmatrix} \nabla_{\gamma_r} \tilde{W}_e \\ \nabla_\theta \tilde{W}_e \end{bmatrix}}_{\text{passivation}}$$

$$\mathcal{Z}_1^\star = \{\boldsymbol{x} : \boldsymbol{\nu} = 0,\ \boldsymbol{\omega} = \omega_0 \mathbb{1}\}$$

consensus zero-dynamics manifold

General strategy

- initial specs: energy bias (lose passivity)
- recover transverse passivity
- design potential energy function (tangential goal)
- stabilize the refined target set



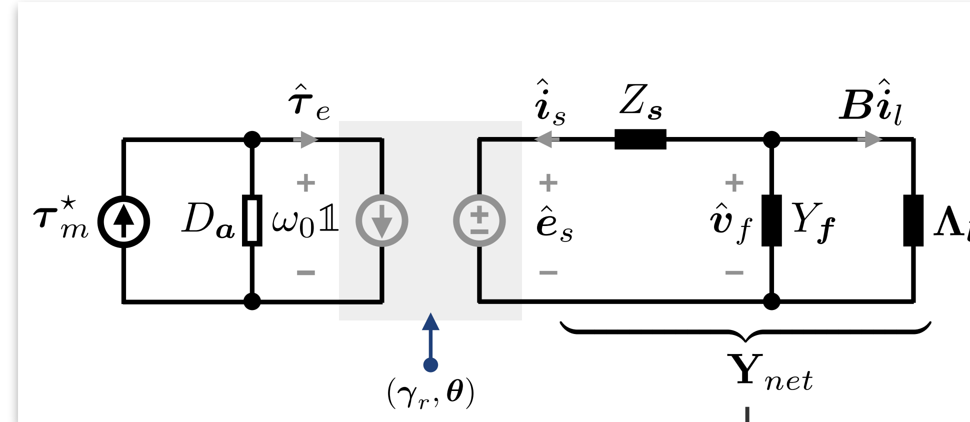
$$\begin{bmatrix} \boldsymbol{\rho}_m \\ \boldsymbol{\tau}_m \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ D_{\boldsymbol{a}}\omega_0\mathbb{1} \end{bmatrix} + \begin{bmatrix} \hat{\boldsymbol{\rho}}_e - \nabla_{\gamma_r} S \\ \hat{\boldsymbol{\tau}}_e - \nabla_{\theta} S \end{bmatrix}}_{\text{controlled-invariance}} - \underbrace{\begin{bmatrix} \nabla_{\gamma_r} \tilde{W}_e \\ \nabla_{\theta} \tilde{W}_e \end{bmatrix}}_{\text{passivation}}$$

$$\mathcal{Z}_2^{\star} = \{\boldsymbol{x} : \boldsymbol{\nu} = 0,\ \boldsymbol{\omega} = \omega_0\mathbb{1},\ \begin{bmatrix} \boldsymbol{i}_s \\ \boldsymbol{v}_f \\ \boldsymbol{i}_l \end{bmatrix} = \boldsymbol{\Pi}\, \underbrace{\boldsymbol{R_\theta} L_{\boldsymbol{m}}(\mathrm{e}^{[\gamma_r]} \otimes \mathrm{g}_2)\omega_0\mathbb{1}}_{\hat{\boldsymbol{e}}_s} \}$$

consensus zero-dynamics manifold | steady-state response

General strategy

- initial specs: energy bias (lose passivity)
- recover transverse passivity
- design potential energy function (tangential goal)
- stabilize the refined target set



$$\mathbf{Y}_{net} = \left(Z_s + (Y_f + \boldsymbol{\Lambda}_l)^{-1}\right)^{-1}$$

$$\boldsymbol{\Lambda}_l = B Z_l^{-1} B^\top$$

$$\begin{bmatrix}\boldsymbol{\rho}_m \\ \boldsymbol{\tau}_m\end{bmatrix} = \underbrace{\begin{bmatrix}0 \\ D_a\omega_0\mathbb{1}\end{bmatrix} + \begin{bmatrix}\hat{\boldsymbol{\rho}}_e - \nabla_{\gamma_r} S \\ \hat{\boldsymbol{\tau}}_e - \nabla_{\theta} S\end{bmatrix}}_{\text{controlled-invariance}} \underbrace{- \begin{bmatrix}\nabla_{\gamma_r}\tilde{W}_e \\ \nabla_{\theta}\tilde{W}_e\end{bmatrix}}_{\text{passivation}}$$

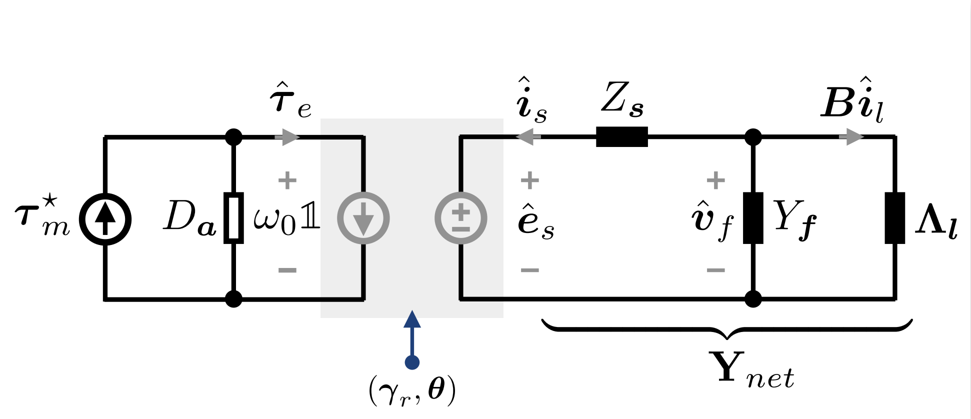
$$\mathcal{Z}_2^\star = \left\{\boldsymbol{x} : \boldsymbol{\nu} = 0,\ \boldsymbol{\omega} = \omega_0\mathbb{1},\ \begin{bmatrix}\boldsymbol{i}_s \\ \boldsymbol{v}_f \\ \boldsymbol{i}_l\end{bmatrix} = \boldsymbol{\Pi}\, \underbrace{\boldsymbol{R}_{\boldsymbol{\theta}} L_{\boldsymbol{m}}(\mathrm{e}^{[\gamma_r]} \otimes \mathrm{g}_2)\omega_0\mathbb{1}}_{\hat{\boldsymbol{e}}_s}\right\}$$

consensus zero-dynamics manifold

steady-state response

$$\boldsymbol{\Pi} = \begin{bmatrix} -(Z_s + (Y_f + \boldsymbol{\Lambda}_l)^{-1})^{-1} \\ (Y_f + \boldsymbol{\Lambda}_t)^{-1}(Z_s + (Y_f + \boldsymbol{\Lambda}_l)^{-1})^{-1} \\ Z_l^{-1} B^\top (Y_f + \boldsymbol{\Lambda}_t)^{-1}(Z_s + (Y_f + \boldsymbol{\Lambda}_l)^{-1})^{-1} \end{bmatrix}$$

General strategy



- initial specs: energy bias (lose passivity)
- recover transverse passivity
- design potential energy function (tangential goal)
- stabilize the refined target set

$$\begin{bmatrix} \boldsymbol{\rho}_m \\ \boldsymbol{\tau}_m \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ D_a \omega_0 \mathbb{1} \end{bmatrix} + \begin{bmatrix} \hat{\boldsymbol{\rho}}_e - \nabla_{\boldsymbol{\gamma}_r} S \\ \hat{\boldsymbol{\tau}}_e - \nabla_{\boldsymbol{\theta}} S \end{bmatrix}}_{\text{controlled-invariance}} - \underbrace{\begin{bmatrix} \nabla_{\boldsymbol{\gamma}_r} \tilde{W}_e \\ \nabla_{\boldsymbol{\theta}} \tilde{W}_e \end{bmatrix}}_{\text{passivation}}$$

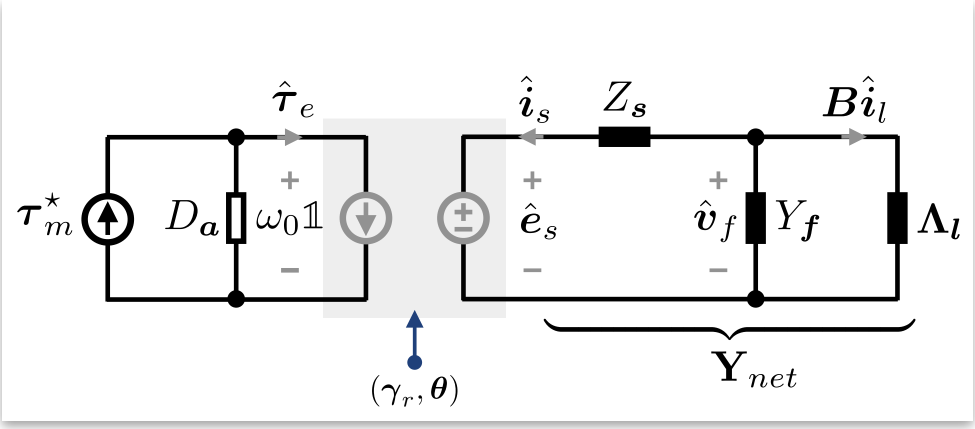
$$\mathcal{Z}_2^\star = \left\{ \boldsymbol{x} : \boldsymbol{\nu} = 0,\ \boldsymbol{\omega} = \omega_0 \mathbb{1},\ \begin{bmatrix} \boldsymbol{i}_s \\ \boldsymbol{v}_f \\ \boldsymbol{i}_l \end{bmatrix} = \boldsymbol{\pi}(\boldsymbol{\gamma}_r, \boldsymbol{\theta}) \right\}$$

consensus zero-dynamics manifold | steady-state response

- where: $\tilde{W}_e = \frac{1}{2}(\boldsymbol{i}_s - \hat{\boldsymbol{i}}_s)^\top L_s (\boldsymbol{i}_s - \hat{\boldsymbol{i}}_s) + \frac{1}{2}(\boldsymbol{v}_f - \hat{\boldsymbol{v}}_f)^\top C_f (\boldsymbol{v}_f - \hat{\boldsymbol{v}}_f) + \frac{1}{2}(\boldsymbol{i}_l - \hat{\boldsymbol{i}}_l)^\top L_l (\boldsymbol{i}_l - \hat{\boldsymbol{i}}_l)$

$$S = \frac{1}{2}\hat{\boldsymbol{i}}_s^\top L_s \hat{\boldsymbol{i}}_s - \frac{1}{2}\hat{\boldsymbol{v}}_f^\top C_f \hat{\boldsymbol{v}}_f + \frac{1}{2}\hat{\boldsymbol{i}}_l^\top L_l \hat{\boldsymbol{i}}_l$$

General strategy



- initial specs: energy bias (lose passivity)
- recover transverse passivity
- design potential energy function (tangential goal)
- stabilize the refined target set

$$\begin{bmatrix}\boldsymbol{\rho}_m \\ \boldsymbol{\tau}_m\end{bmatrix} = \underbrace{\begin{bmatrix}0 \\ D_{\boldsymbol{a}}\omega_0\mathbb{1}\end{bmatrix} + \begin{bmatrix}\hat{\boldsymbol{\rho}}_e - \nabla_{\boldsymbol{\gamma}_r}\mathcal{S} \\ \hat{\boldsymbol{\tau}}_e - \nabla_{\boldsymbol{\theta}}\mathcal{S}\end{bmatrix}}_{\text{controlled-invariance}} \underbrace{- \begin{bmatrix}\nabla_{\boldsymbol{\gamma}_r}\tilde{W}_{\boldsymbol{e}} \\ \nabla_{\boldsymbol{\theta}}\tilde{W}_{\boldsymbol{e}}\end{bmatrix}}_{\text{passivation}}$$

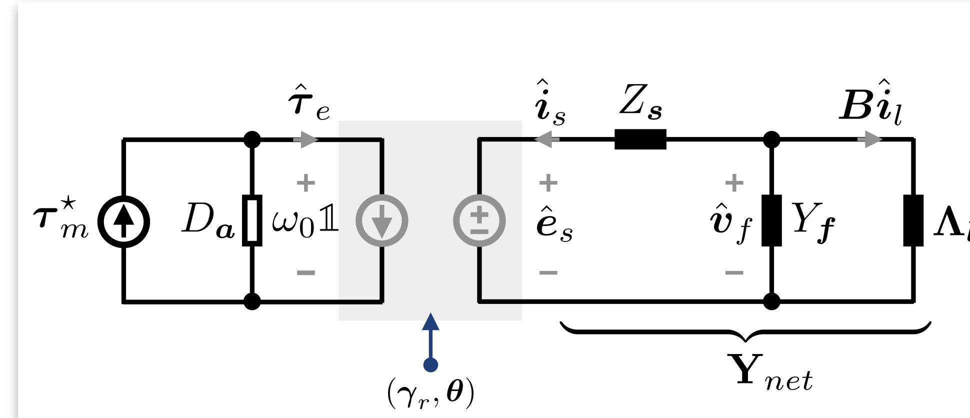
$$\mathcal{Z}_3^\star = \left\{\boldsymbol{x} : \boldsymbol{\nu} = 0,\ \boldsymbol{\omega} = \omega_0\mathbb{1},\ \begin{bmatrix}\boldsymbol{i}_s \\ \boldsymbol{v}_f \\ \boldsymbol{i}_l\end{bmatrix} = \boldsymbol{\pi}(\boldsymbol{\gamma}_r, \boldsymbol{\theta}),\ \nabla\mathcal{S}(\boldsymbol{\gamma}_r, \boldsymbol{\theta}) = 0\right\}$$

consensus zero-dynamics manifold | steady-state response | potential energy shaping

- where: $\tilde{W}_{\boldsymbol{e}} = \frac{1}{2}(\boldsymbol{i}_s - \hat{\boldsymbol{i}}_s)^\top L_{\boldsymbol{s}}(\boldsymbol{i}_s - \hat{\boldsymbol{i}}_s) + \frac{1}{2}(\boldsymbol{v}_f - \hat{\boldsymbol{v}}_f)^\top C_{\boldsymbol{f}}(\boldsymbol{v}_f - \hat{\boldsymbol{v}}_f) + \frac{1}{2}(\boldsymbol{i}_l - \hat{\boldsymbol{i}}_l)^\top L_{\boldsymbol{l}}(\boldsymbol{i}_l - \hat{\boldsymbol{i}}_l)$

$$\mathcal{S} = \frac{1}{2}\hat{\boldsymbol{i}}_s^\top L_{\boldsymbol{s}}\hat{\boldsymbol{i}}_s - \frac{1}{2}\hat{\boldsymbol{v}}_f^\top C_{\boldsymbol{f}}\hat{\boldsymbol{v}}_f + \frac{1}{2}\hat{\boldsymbol{i}}_l^\top L_{\boldsymbol{l}}\hat{\boldsymbol{i}}_l$$

General strategy



- initial specs: energy bias (lose passivity)
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$$\begin{bmatrix} \boldsymbol{\rho}_m \\ \boldsymbol{\tau}_m \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ D_a \omega_0 \mathbb{1} \end{bmatrix} + \begin{bmatrix} \hat{\boldsymbol{\rho}}_e - \nabla_{\gamma_r}\mathcal{S} \\ \hat{\boldsymbol{\tau}}_e - \nabla_{\boldsymbol{\theta}}\mathcal{S} \end{bmatrix}}_{\text{controlled-invariance}} + \underbrace{\cdots}_{\text{passivation}}$$

accounts for $\mathfrak{Re}\{\mathbf{Y}_{net}\}$

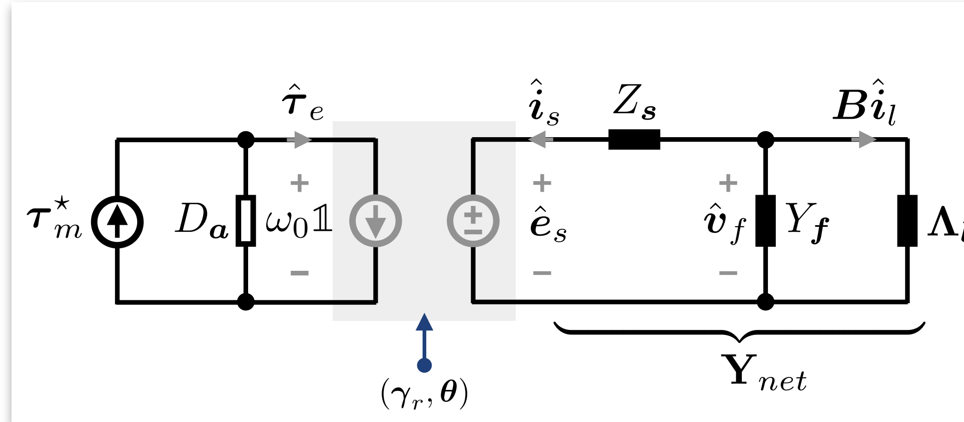
$$\mathcal{Z}_3^\star = \left\{ \boldsymbol{x} : \boldsymbol{\nu} = 0,\ \boldsymbol{\omega} = \omega_0 \mathbb{1},\ \begin{bmatrix} \boldsymbol{i}_s \\ \boldsymbol{v}_f \\ \boldsymbol{i}_l \end{bmatrix} = \boldsymbol{\pi}(\gamma_r, \boldsymbol{\theta}),\ \nabla\mathcal{S}(\gamma_r, \boldsymbol{\theta}) = 0 \right\}$$

consensus zero-dynamics manifold | steady-state response | potential energy shaping

- where: $\tilde{W}_e = \frac{1}{2}(\boldsymbol{i}_s - \hat{\boldsymbol{i}}_s)^\top L_s (\boldsymbol{i}_s - \hat{\boldsymbol{i}}_s) + \frac{1}{2}(\boldsymbol{v}_f - \hat{\boldsymbol{v}}_f)^\top C_f (\boldsymbol{v}_f - \hat{\boldsymbol{v}}_f) + \frac{1}{2}(\boldsymbol{i}_l - \hat{\boldsymbol{i}}_l)^\top L_l (\boldsymbol{i}_l - \hat{\boldsymbol{i}}_l)$

$$\mathcal{S} = \frac{1}{2}\hat{\boldsymbol{i}}_s^\top L_s \hat{\boldsymbol{i}}_s - \frac{1}{2}\hat{\boldsymbol{v}}_f^\top C_f \hat{\boldsymbol{v}}_f + \frac{1}{2}\hat{\boldsymbol{i}}_l^\top L_l \hat{\boldsymbol{i}}_l$$

General strategy



- initial specs: energy bias (lose passivity)
- recover transverse passivity
- design potential energy function (tangential goal)
- stabilize the refined target set

$$\begin{bmatrix} -\nabla_{\gamma_r}\mathcal{S} \\ -\nabla_{\theta}\mathcal{S} \end{bmatrix}$$

synchronization torque

controlled-invariance

passivation

$$\begin{bmatrix} -\frac{\alpha_l}{\alpha_l^2+\omega_0^2}\hat{P}_l - \frac{\omega_0}{\alpha_l^2+\omega_0^2}\hat{Q}_l \\ -\frac{\omega_0}{\alpha_l^2+\omega_0^2}\hat{P}_l + \frac{\alpha_l}{\alpha_l^2+\omega_0^2}\hat{Q}_l \end{bmatrix}$$

$$\mathcal{Z}_3^\star = \left\{\boldsymbol{x} : \boldsymbol{\nu} = 0,\ \boldsymbol{\omega} = \omega_0\mathbb{1},\ \begin{bmatrix} \boldsymbol{i}_s \\ \boldsymbol{v}_f \\ \boldsymbol{i}_l \end{bmatrix} = \boldsymbol{\pi}(\boldsymbol{\gamma}_r, \boldsymbol{\theta}),\ \nabla\mathcal{S}(\boldsymbol{\gamma}_r, \boldsymbol{\theta}) = 0\right\}$$

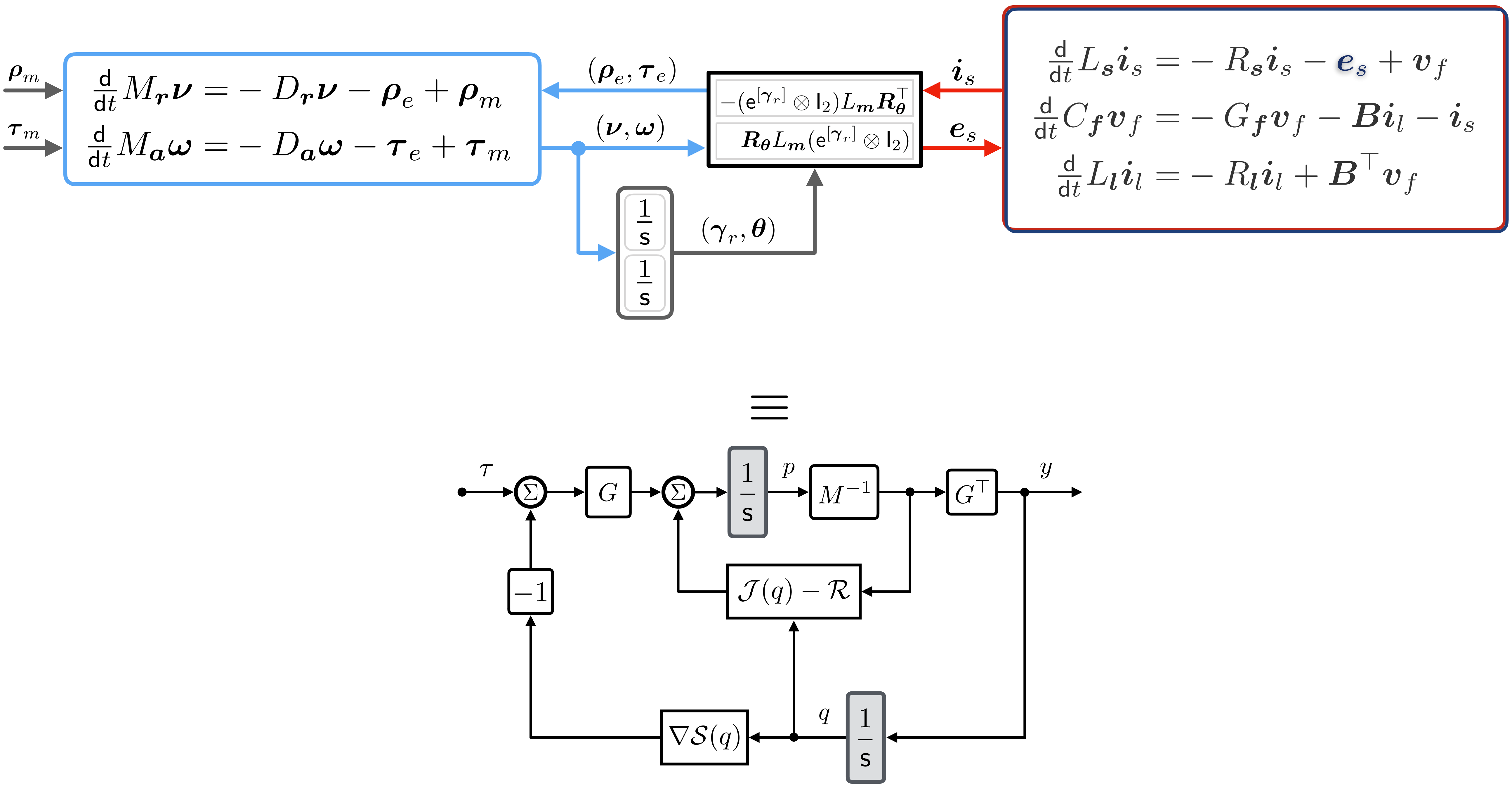
consensus zero-dynamics manifold | steady-state response | potential energy shaping

- where: $\tilde{W}_e = \frac{1}{2}(\boldsymbol{i}_s - \hat{\boldsymbol{i}}_s)^\top L_s(\boldsymbol{i}_s - \hat{\boldsymbol{i}}_s) + \frac{1}{2}(\boldsymbol{v}_f - \hat{\boldsymbol{v}}_f)^\top C_f(\boldsymbol{v}_f - \hat{\boldsymbol{v}}_f) + \frac{1}{2}(\boldsymbol{i}_l - \hat{\boldsymbol{i}}_l)^\top L_l(\boldsymbol{i}_l - \hat{\boldsymbol{i}}_l)$

$$\mathcal{S} = \tfrac{1}{2}\hat{\boldsymbol{i}}_s^\top L_s\hat{\boldsymbol{i}}_s - \tfrac{1}{2}\hat{\boldsymbol{v}}_f^\top C_f\hat{\boldsymbol{v}}_f + \tfrac{1}{2}\hat{\boldsymbol{i}}_l^\top L_l\hat{\boldsymbol{i}}_l$$

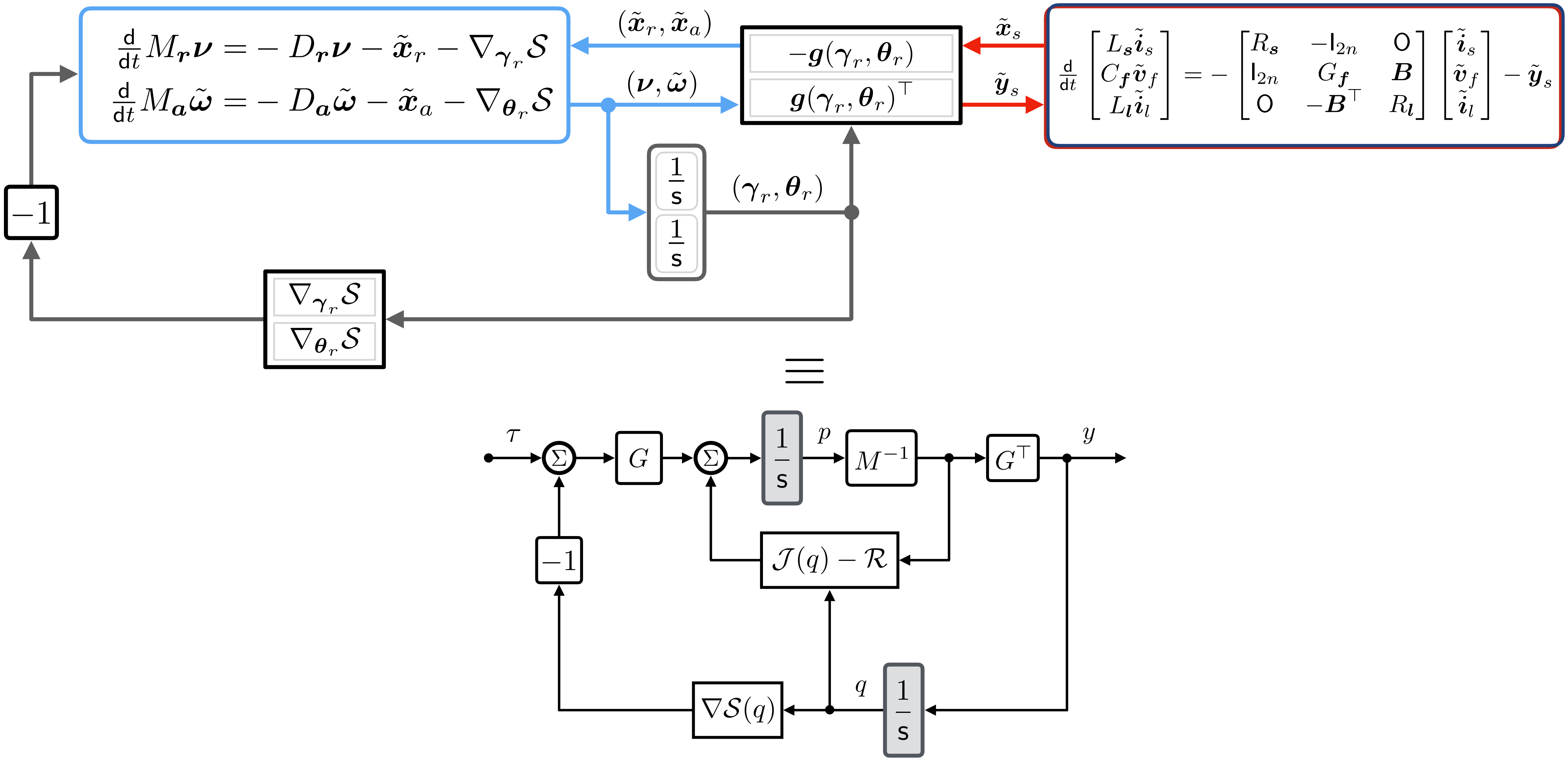
Power system dynamics

- passive with $\mathcal{H} = \frac{1}{2}\boldsymbol{\nu}^\top M_{\boldsymbol{r}}\boldsymbol{\nu} + \frac{1}{2}\boldsymbol{\omega}^\top M_{\boldsymbol{a}}\boldsymbol{\omega} + \frac{1}{2}\boldsymbol{i}_s^\top L_{\boldsymbol{s}}\boldsymbol{i}_s + \frac{1}{2}\boldsymbol{v}_f^\top C_{\boldsymbol{f}}\boldsymbol{v}_f + \frac{1}{2}\boldsymbol{i}_l^\top L_{\boldsymbol{l}}\boldsymbol{i}_l$
- zero potential energy



Transient dynamics

- passive with $\tilde{\mathcal{H}} = \frac{1}{2}\boldsymbol{\nu}^\top M_{\boldsymbol{r}}\boldsymbol{\nu} + \frac{1}{2}\tilde{\boldsymbol{\omega}}^\top M_{\boldsymbol{a}}\tilde{\boldsymbol{\omega}} + \frac{1}{2}\tilde{\boldsymbol{i}}_s^\top L_{\boldsymbol{s}}\tilde{\boldsymbol{i}}_s + \frac{1}{2}\tilde{\boldsymbol{v}}_f^\top C_{\boldsymbol{f}}\tilde{\boldsymbol{v}}_f + \frac{1}{2}\tilde{\boldsymbol{i}}_l^\top L_{\boldsymbol{l}}\tilde{\boldsymbol{i}}_l + \mathcal{S}(\boldsymbol{\gamma}_r, \boldsymbol{\theta}_r)$
- different energy exchange structure



Concluding thoughts

- Relationship between active and reactive power and the ray-circle complementarity
- Control of the modulated energy transfer done via dynamic extension
- Zero-dynamics refinement helps construct transverse coordinates
- Circuit potential energy reveal inherent synchronization and optimal network flow

Related publications

C. Arghir and F. Dörfler. "Energy-based Stabilization of Network Flows in Multi-machine Power Systems". In: *Proceedings of the 23rd International Symposium on Mathematical Theory of Networks and Systems*. July 2018.

C. Arghir. "Transverse feedback passivation in control of multi-machine and multi-converter power networks". In: *ETH Research Collection, Doctoral Thesis*. June 2020.

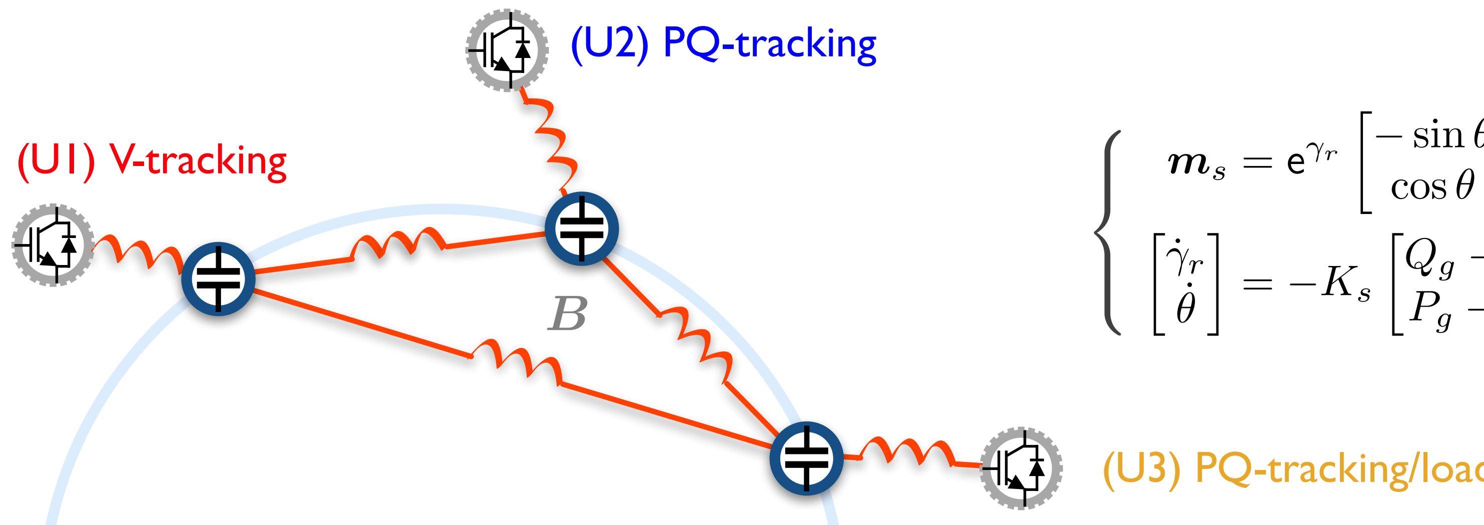
ETH zürich

Thank you

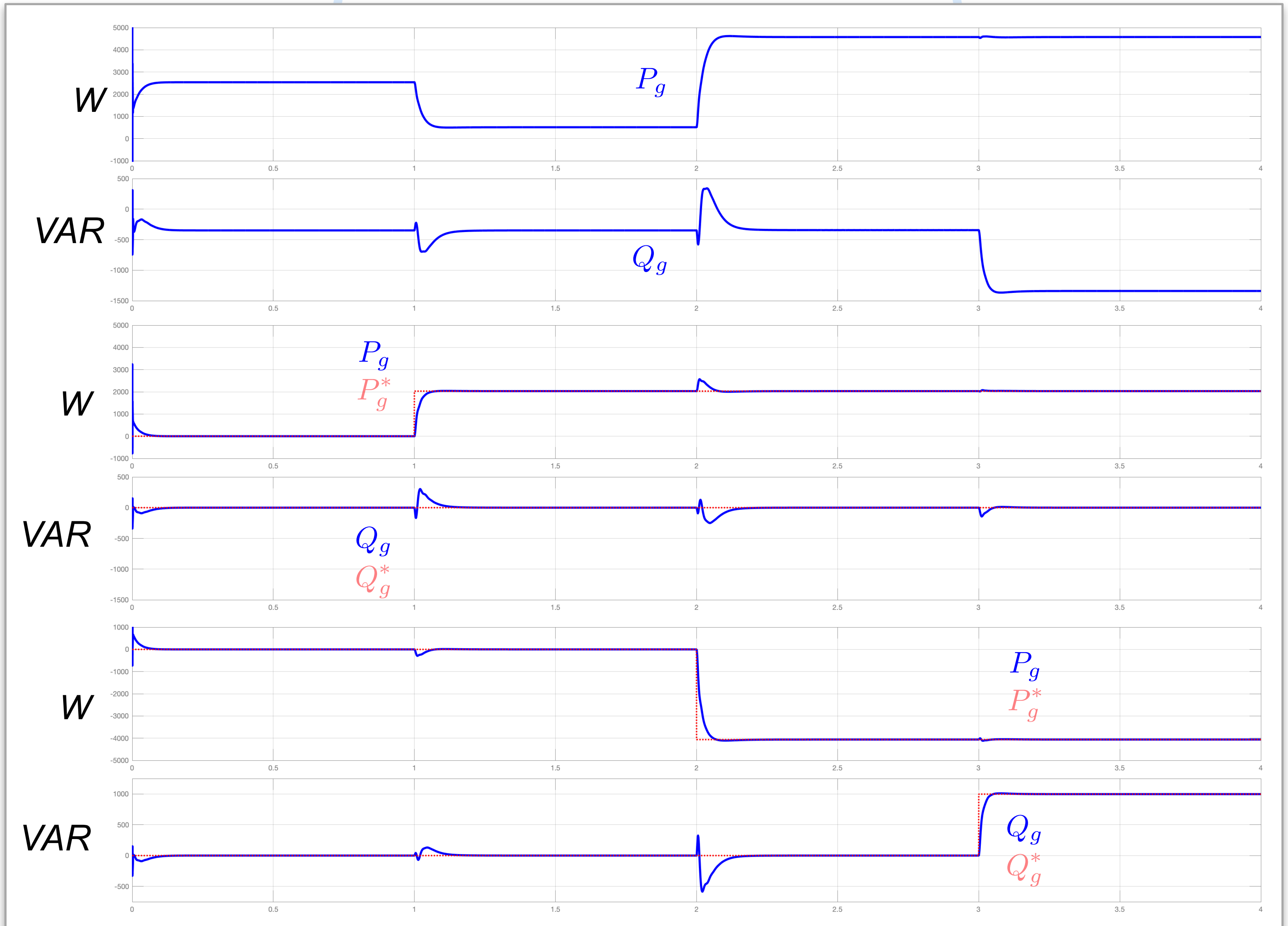
Part V
Additional material

Test case

$$\begin{cases} \boldsymbol{m}_s = \mathrm{e}^{\gamma_r} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \\ \dot{\gamma}_r = -K_f(\|\boldsymbol{v}_f\|^2 - v_f^{*2}) \\ \dot{\theta} = \eta v_{dc} \end{cases}$$

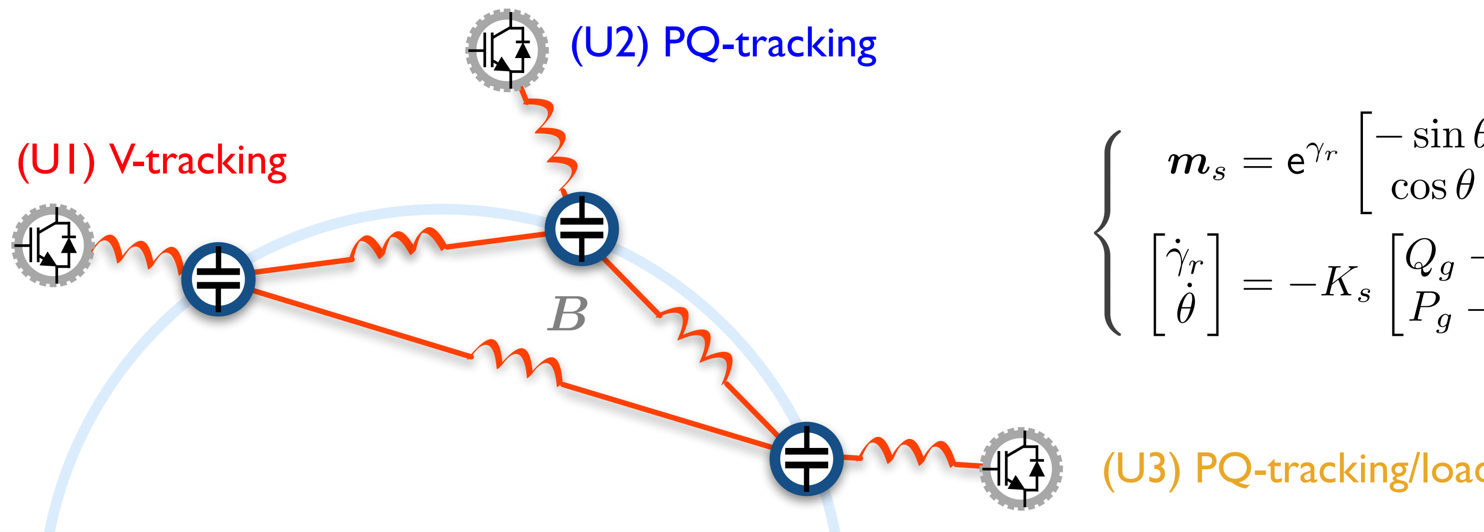


$$\begin{cases} \boldsymbol{m}_s = \mathrm{e}^{\gamma_r} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \\ \begin{bmatrix} \dot{\gamma}_r \\ \dot{\theta} \end{bmatrix} = -K_s \begin{bmatrix} Q_g - Q_g^* \\ P_g - P_g^* \end{bmatrix} + \begin{bmatrix} 0 \\ \eta v_{dc} \end{bmatrix} \end{cases}$$

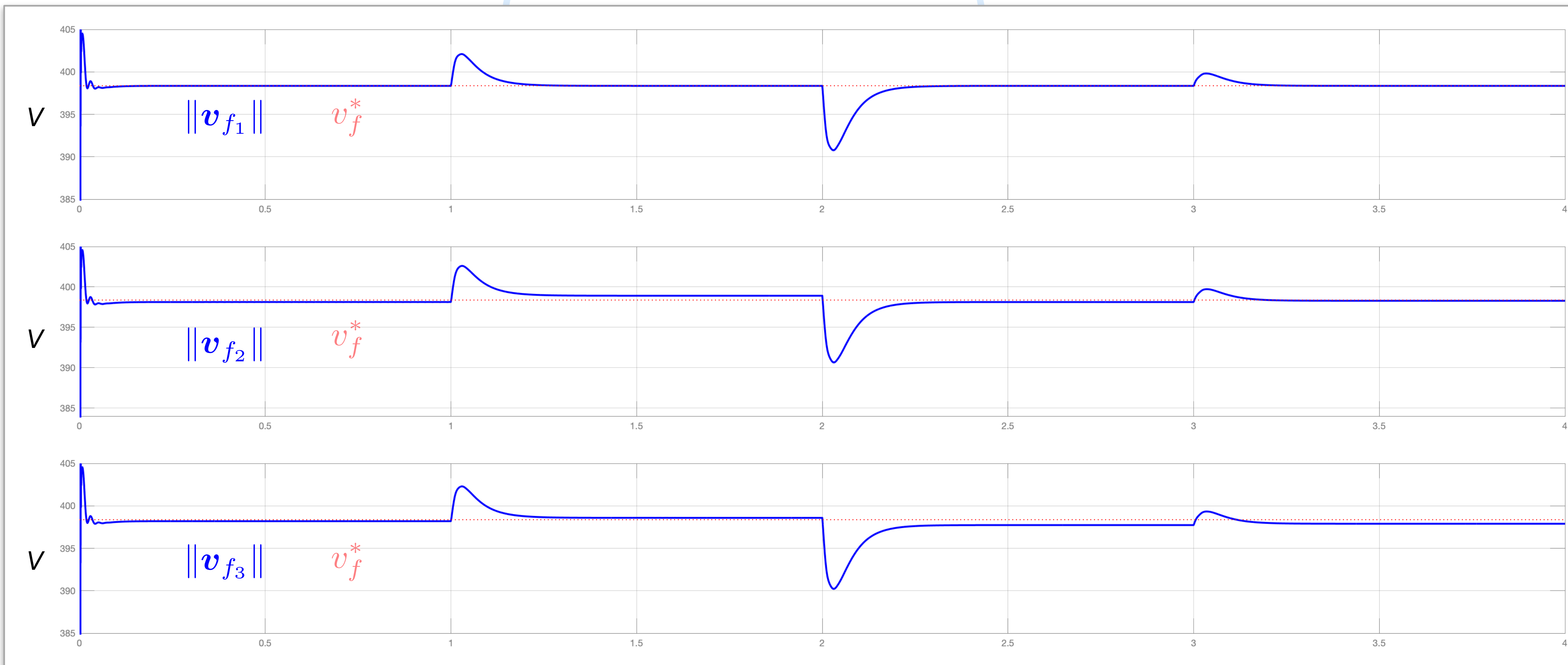


Test case

$$\begin{cases} \boldsymbol{m}_s = \mathrm{e}^{\gamma_r} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \\ \dot{\gamma}_r = -K_f(\|\boldsymbol{v}_f\|^2 - v_f^{*2}) \\ \dot{\theta} = \eta v_{dc} \end{cases}$$

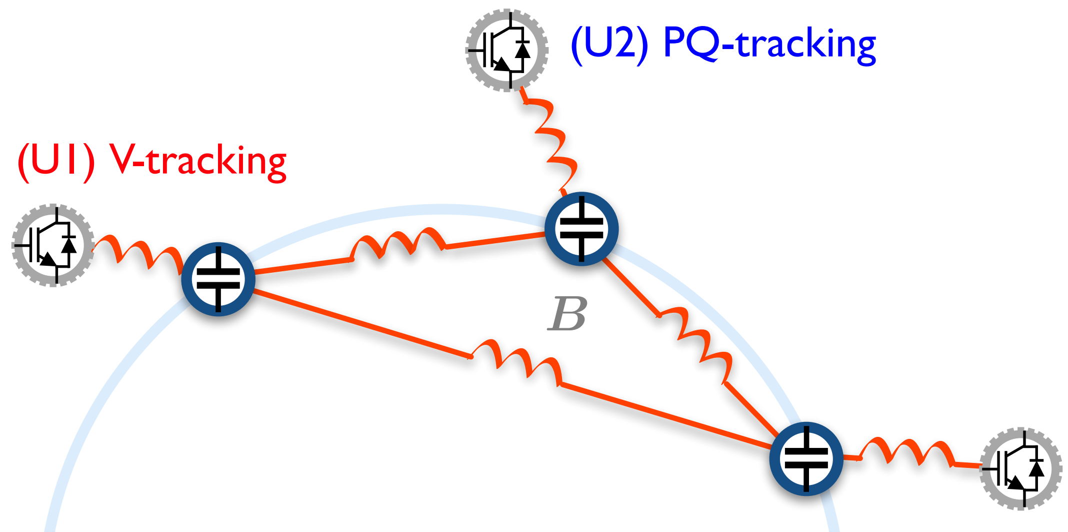


$$\begin{cases} \boldsymbol{m}_s = \mathrm{e}^{\gamma_r} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \\ \begin{bmatrix} \dot{\gamma}_r \\ \dot{\theta} \end{bmatrix} = -K_s \begin{bmatrix} Q_g - Q_g^* \\ P_g - P_g^* \end{bmatrix} + \begin{bmatrix} 0 \\ \eta v_{dc} \end{bmatrix} \end{cases}$$

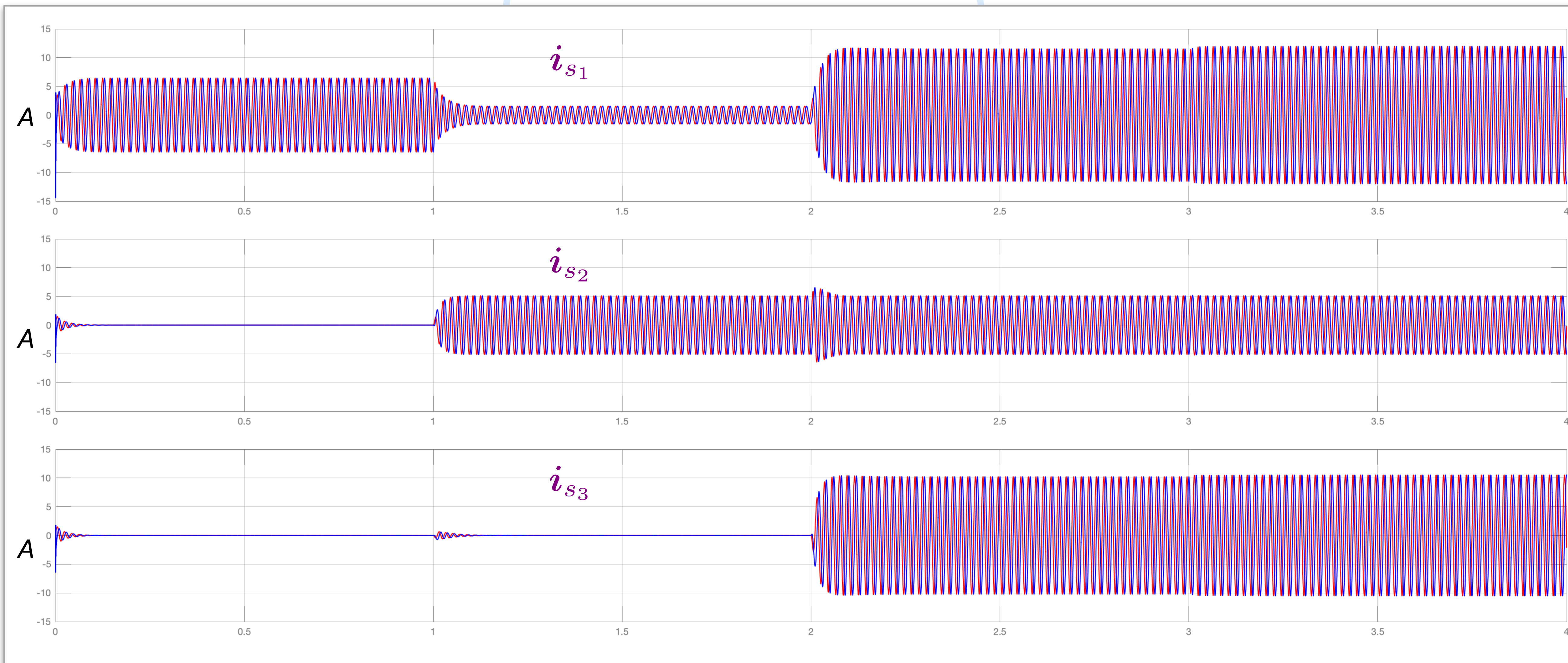


Test case

$$\begin{cases} \boldsymbol{m}_s = \mathrm{e}^{\gamma_r} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \\ \dot{\gamma}_r = -K_f(\|\boldsymbol{v}_f\|^2 - v_f^{*2}) \\ \dot{\theta} = \eta v_{dc} \end{cases}$$



$$\begin{cases} \boldsymbol{m}_s = \mathrm{e}^{\gamma_r} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \\ \begin{bmatrix} \dot{\gamma}_r \\ \dot{\theta} \end{bmatrix} = -K_s \begin{bmatrix} Q_g - Q_g^* \\ P_g - P_g^* \end{bmatrix} + \begin{bmatrix} 0 \\ \eta v_{dc} \end{bmatrix} \end{cases}$$



Test case

$$\begin{cases} \boldsymbol{m}_s = \mathrm{e}^{\gamma_r} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \\ \dot{\gamma}_r = -K_f(\|\boldsymbol{v}_f\|^2 - v_f^{*2}) \\ \dot{\theta} = \eta v_{dc} \end{cases}$$

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